Chapter 17, Solution 1.

(a)   This is **periodic** with $\omega = \pi$ which leads to $T = 2\pi/\omega = $ **2**.

(b)   y(t) is **not periodic** although sin t and 4 cos 2πt are independently periodic.

(c)   Since $\sin A \cos B = 0.5[\sin(A + B) + \sin(A - B)]$,
            g(t) = sin 3t cos 4t = 0.5[sin 7t + sin(–t)] = –0.5 sin t + 0.5 sin7t
        which is harmonic or **periodic** with the fundamental frequency
                    $\omega = 1$ or $T = 2\pi/\omega = $ **2π.**

(d)   h(t) = $\cos^2 t$ = 0.5(1 + cos 2t). Since the sum of a periodic function and a constant is also **periodic**, h(t) is periodic. $\omega = 2$ or $T = 2\pi/\omega = $ **π.**

(e)   The frequency ratio $0.6|0.4 = 1.5$ makes z(t) **periodic**.
                    $\omega = 0.2\pi$ or $T = 2\pi/\omega = $ **10**.

(f)   p(t) = 10 is **not periodic**.

(g)   g(t) is **not periodic**.

Chapter 17, Solution 2.

(a)   The frequency ratio is $6/5 = 1.2$. The highest common factor is 1.
                    $\omega = 1 = 2\pi/T$ or $T = $ **2π**.

(b) $\omega = 2$ or $T = 2\pi/\omega = $ **π**.

(c) $f_3(t) = 4 \sin^2 600\pi t = (4/2)(1 - \cos 1200\pi t)$
                    $\omega = 1200\pi$ or $T = 2\pi/\omega = 2\pi/(1200\pi) = $ **1/600**.

(d) $f_4(t) = e^{j10t} = \cos 10t + j\sin 10t$. $\omega = 10$ or $T = 2\pi/\omega = $ **0.2π**.

Chapter 17, Solution 3.

$$T = 4, \quad \omega_o = 2\pi/T = \pi/2$$

$$g(t) = \begin{array}{ll} 5, & 0 < t < 1 \\ 10, & 1 < t < 2 \\ 0, & 2 < t < 4 \end{array}$$

$$a_o = (1/T)\int_0^T g(t)dt = 0.25[\int_0^1 5dt + \int_1^2 10dt] = \underline{\mathbf{3.75}}$$

$$a_n = (2/T) \int_0^T g(t)\cos(n\omega_o t)dt = (2/4)[\int_0^1 5\cos(\frac{n\pi}{2}t)dt + \int_1^2 10\cos(\frac{n\pi}{2}t)dt]$$

$$= 0.5[5\frac{2}{n\pi}\sin\frac{n\pi}{2}t\Big|_0^1 + 10\frac{2}{n\pi}\sin\frac{n\pi}{2}t\Big|_1^2] = (-1/(n\pi))5\sin(n\pi/2)$$

$$a_n = \begin{array}{ll} \mathbf{(5/(n\pi))(-1)^{(n+1)/2}}, & \mathbf{n = odd} \\ \mathbf{0,} & \mathbf{n = even} \end{array}$$

$$b_n = (2/T) \int_0^T g(t)\sin(n\omega_o t)dt = (2/4)[\int_0^1 5\sin(\frac{n\pi}{2}t)dt + \int_1^2 10\sin(\frac{n\pi}{2}t)dt]$$

$$= 0.5[\frac{-2x5}{n\pi}\cos\frac{n\pi}{2}t\Big|_0^1 - \frac{2x10}{n\pi}\cos\frac{n\pi}{2}t\Big|_1^2] = \mathbf{(5/(n\pi))[3 - 2\cos n\pi + \cos(n\pi/2)]}$$

Chapter 17, Solution 4.

$$f(t) = 10 - 5t, \quad 0 < t < 2, \quad T = 2, \quad \omega_o = 2\pi/T = \pi$$

$$a_o = (1/T) \int_0^T f(t)dt = (1/2)\int_0^2 (10 - 5t)dt = 0.5[10t - (5t^2/2)]\Big|_0^2 = 5$$

$$a_n = (2/T) \int_0^T f(t)\cos(n\omega_o t)dt = (2/2)\int_0^2 (10 - 5t)\cos(n\pi t)dt$$

$$= \int_0^2 (10)\cos(n\pi t)dt - \int_0^2 (5t)\cos(n\pi t)dt$$

$$= \frac{-5}{n^2\pi^2}\cos n\pi t\Big|_0^2 + \frac{5t}{n\pi}\sin n\pi t\Big|_0^2 = [-5/(n^2\pi^2)](\cos 2n\pi - 1) = 0$$

$$b_n = (2/2) \int_0^2 (10 - 5t) \sin(n\pi t)dt$$

$$= \int_0^2 (10) \sin(n\pi t)dt - \int_0^2 (5t) \sin(n\pi t)dt$$

$$= \frac{-5}{n^2\pi^2} \sin n\pi t \Big|_0^2 + \frac{5t}{n\pi} \cos n\pi t \Big|_0^2 = 0 + [10/(n\pi)](\cos 2n\pi) = 10/(n\pi)$$

Hence $\qquad f(t) = \mathbf{5 + \dfrac{10}{\pi} \sum_{n=1}^{\infty} \dfrac{1}{n} \sin(n\pi t)}$

Chapter 17, Solution 5.

$$T = 2\pi, \qquad \omega = 2\pi/T = 1$$

$$a_o = \frac{1}{T} \int_0^T z(t)dt = \frac{1}{2\pi}[1x\pi - 2x\pi] = -0.5$$

$$a_n = \frac{2}{T} \int_0^T z(t) \cos n\omega_o dt = \frac{1}{\pi} \int_0^\pi 1\cos n t\, dt - \frac{1}{\pi} \int_\pi^{2\pi} 2\cos n t\, dt = \frac{1}{n\pi} \sin ..nt \Big|_0^\pi - \frac{2}{n\pi} \sin nt \Big|_\pi^{2\pi} = 0$$

$$b_n = \frac{2}{T} \int_0^T z(t) \cos n\omega_o dt = \frac{1}{\pi} \int_0^\pi 1\sin n t\, dt - \frac{1}{\pi} \int_\pi^{2\pi} 2\sin n t\, dt = -\frac{1}{n\pi} \cos nt \Big|_0^\pi + \frac{2}{n\pi} \cos nt \Big|_\pi^{2\pi} = \begin{cases} \dfrac{6}{n\pi}, n = \text{odd} \\ 0, \quad n = \text{even} \end{cases}$$

Thus,

$$z(t) = -0.5 + \sum_{\substack{n=1}}^{\infty} \frac{6}{n\pi} \sin nt$$

$$n=\text{odd}$$

Chapter 17, Solution 6.

$$T = 2, \ \omega_o = \frac{2\pi}{2} = \pi$$

$$a_o = \frac{1}{2}\int_0^2 y(t)dt = \frac{1}{2}(4x1 + 2x1) = \frac{6}{2} = 3$$

Since this is an odd function, $a_n = 0$.

$$b_n = \frac{2}{2}\int_0^2 y(t)\sin(n\omega_o t)dt = \int_0^1 4\sin(n\pi t)dt + \int_1^2 2\sin(n\pi t)dt$$

$$= \frac{-4}{n\pi}\cos(n\pi t)\Big|_0^1 - \frac{2}{n\pi}\cos(n\pi t)\Big|_1^2 = \frac{-4}{n\pi}(\cos(n\pi) - 1) - \frac{2}{n\pi}(\cos(2n\pi) - \cos(n\pi))$$

$$= \frac{4}{n\pi}(1 - \cos(n\pi)) - \frac{2}{n\pi}(1 - \cos(n\pi)) = \frac{2}{n\pi}(1 - \cos(n\pi)) = \begin{vmatrix} 0, & n=\text{even} \\ \dfrac{4}{n\pi}, & n=\text{odd} \end{vmatrix}$$

$$y(t) = 3 + \frac{4}{\pi}\sum_{\substack{n=1 \\ n=\text{odd}}}^{\infty} \frac{1}{n}\sin(n\pi t)$$

Chapter 17, Solution 7.

$$T = 12, \quad \omega = 2\pi/T = \frac{\pi}{6}, \quad a_0 = 0$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_o dt = \frac{1}{6}[\int_{-2}^4 10\cos n\pi t/6 dt + \int_4^{10}(-10)\cos n\pi t/6 dt]$$

$$= \frac{10}{n\pi}\sin n\pi t/6 \ \Big|_{-2}^4 - \frac{10}{n\pi}\sin n\pi t/6 \ \Big|_4^{10} = \frac{10}{n\pi}\Big[2\sin 2n\pi/3 + \sin n\pi/3 - \sin 5n\pi/3\Big]$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_o dt = \frac{1}{6}[\int_{-2}^4 10\sin n\pi t/6 dt + \int_4^{10}(-10)\sin n\pi t/6 dt]$$

$$= -\frac{10}{n\pi}\cos n\pi t/6 \Big|_{-2}^{4} + \frac{10}{n\pi}\cos n\pi nt/6 \Big|_{4}^{10} = \frac{10}{n\pi}\left[\cos 5n\pi/3 + \cos n\pi/3 - 2\sin 2n\pi/3\right]$$

$$f(t) = \sum_{n=1}^{\infty}\left(a_n \cos n\pi t/6 + b_n \sin n\pi t/6\right)$$

where a_n and b_n are defined above.

Chapter 17, Solution 8.

$f(t) = 2(1+t), \; -1 < t < 1, \quad T = 2, \quad \omega_0 = 2\pi/T = \pi$

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{2}\int_{-1}^{1} 2(t+1)dt = t^2 + t \Big|_{-1}^{1} = 2$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega_0 dt = \frac{2}{2}\int_{-1}^{1} 2(t+1)\cos n\pi t\, dt = 2\left(\frac{1}{n^2\pi^2}\cos n\pi t + \frac{t}{n\pi}\sin n\pi t + \frac{1}{n\pi}\sin n\pi t\right)\Big|_{-1}^{1} = 0$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega_0 dt = \frac{2}{2}\int_{-1}^{1} 2(t+1)\sin n\pi t\, dt = 2\left(-\frac{1}{n^2\pi^2}\sin n\pi t - \frac{t}{n\pi}\cos n\pi t - \frac{1}{n\pi}\cos n\pi t\right)\Big|_{-1}^{1} = -\frac{4}{n\pi}\cos n\pi$$

$$f(t) = 2 - \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\cos n\pi t$$

Chapter 17, Solution 9.

f(t) is an even function, $b_n = 0$.

$T = 8, \quad \omega = 2\pi/T = \pi/4$

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{2}{8}\left[\int_0^2 10\cos \pi t/4\, dt + 0\right] = \frac{10}{4}(\frac{4}{\pi})\sin \pi t/4 \Big|_0^2 = \frac{10}{\pi} = 3.183$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos n\omega_o dt = \frac{40}{8}[\int_0^2 10\cos \pi t/4\cos n\pi t/4 dt + 0] = 5\int_0^2 [\cos \pi t(n+1)/4 + \cos \pi t(n-1)/4] dt$$

For n = 1,

$$a_1 = 5\int_0^2 [\cos \pi t/2 + 1] dt = 5\left[\frac{2}{\pi}\sin \pi t/2 dt + t\right]_0^2 = 10$$

For n>1,

$$a_n = \frac{20}{\pi(n+1)}\sin\frac{\pi(n+1)t}{4} + \frac{20}{\pi(n-1)}\sin\frac{\pi(n-1)}{4}\Bigg|_0^2 = \frac{20}{\pi(n+1)}\sin\frac{\pi(n+1)}{2} + \frac{20}{\pi(n-1)}\sin\frac{\pi(n-1)}{2}$$

$$a_2 = \frac{10}{\pi}\sin \pi + \frac{20}{\pi}\sin \pi/2 = 6.3662, \qquad a_3 = \frac{20}{4\pi}\sin 2\pi + \frac{10}{\pi}\sin \pi = 0$$

Thus,

$$\underline{a_0 = 3.183, \qquad a_1 = 10, \qquad a_2 = 6.362, \qquad a_3 = 0, \qquad b_1 = 0 = b_2 = b_3}$$

Chapter 17, Solution 10.

$$T = 2, \qquad \omega_0 = 2\pi/T = \pi$$

$$c_n = \frac{1}{T}\int_0^T h(t)e^{-jn\omega_o t} dt = \frac{1}{2}\left[\int_0^1 4e^{-jn\pi t} dt + \int_1^2 (-2)e^{-jn\pi t} dt\right] = \frac{1}{2}\left[\frac{4e^{-jn\pi t}}{-jn\pi}\Big|_0^1 - \frac{2e^{-jn\pi t}}{-jn\pi}\Big|_1^2\right]$$

$$c_n = \frac{j}{2n\pi}\left[4e^{-j\pi n} - 4 - 2e^{-j2n\pi} + 2e^{-jn\pi}\right] = \frac{j}{2n\pi}[6\cos n\pi - 6] = \begin{cases} -\dfrac{6j}{n\pi}, & n = odd \\ 0, & n = even \end{cases}$$

Thus,

$$f(t) = \sum_{\substack{n=-\infty \\ \underline{n=odd}}}^{\infty} \left(\frac{-j6}{n\pi}\right)e^{jn\pi t}$$

Chapter 17, Solution 11.

$$T = 4, \qquad \omega_0 = 2\pi/T = \pi/2$$

$$c_n = \frac{1}{T}\int_0^T y(t)e^{-jn\omega_o t}\,dt = \frac{1}{4}\left[\int_{-1}^0 (t+1)e^{-jn\pi t/2}\,dt + \int_0^1 (1)e^{-jn\pi t/2}\,dt\right]$$

$$c_n = \frac{1}{4}\left[\frac{e^{-jn\pi t/2}}{-n^2\pi^2/4}(-jn\pi t/2 - 1) - \frac{2}{jn\pi}e^{-jn\pi t/2}\Big|_{-1}^0 - \frac{2}{jn\pi}e^{-jn\pi t/2}\Big|_0^1\right]$$

$$= \frac{1}{4}\left[\frac{4}{n^2\pi^2} - \frac{2}{jn\pi} + \frac{4}{n^2\pi^2}e^{jn\pi/2}(jn\pi/2 - 1) + \frac{2}{jn\pi}e^{jn\pi/2} - \frac{2}{jn\pi}e^{-jn\pi/2} + \frac{2}{jn\pi}\right]$$

But

$$e^{jn\pi/2} = \cos n\pi/2 + j\sin n\pi/2 = j\sin n\pi/2, \qquad e^{-jn\pi/2} = \cos n\pi/2 - j\sin n\pi/2 = -j\sin n\pi/2$$

$$c_n = \frac{1}{n^2\pi^2}\left[1 + j(jn\pi/2 - 1)\sin n\pi/2 + n\pi\sin n\pi/2\right]$$

$$y(t) = \sum_{n=-\infty}^{\infty} \frac{1}{n^2\pi^2}\left[1 + j(jn\pi/2 - 1)\sin n\pi/2 + n\pi\sin n\pi/2\right]e^{jn\pi t/2}$$

Chapter 17, Solution 12.

A voltage source has a periodic waveform defined over its period as
$v(t) = t(2\pi - t)$ V,   for all $0 < t < 2\pi$

Find the Fourier series for this voltage.

$v(t) = 2\pi t - t^2,\ 0 < t < 2\pi,\ T = 2\pi,\ \omega_o = 2\pi/T = 1$

$a_o =$

$$(1/T)\int_0^T f(t)\,dt = \frac{1}{2\pi}\int_0^{2\pi}(2\pi t - t^2)\,dt = \frac{1}{2\pi}(\pi t^2 - t^3/3)\Big|_0^{2\pi} = \frac{4\pi^3}{2\pi}(1 - 2/3) = \frac{2\pi^2}{3}$$

$$a_n = \frac{2}{T}\int_0^T (2\pi t - t^2)\cos(nt)dt = \frac{1}{\pi}\left[\frac{2\pi}{n^2}\cos(nt) + \frac{2\pi t}{n}\sin(nt)\right]\Big|_0^{2\pi}$$

$$-\frac{1}{\pi n^3}\left[2nt\cos(nt) - 2\sin(nt) + n^2t^2\sin(nt)\right]\Big|_0^{2\pi}$$

$$= \frac{2}{n^2}(1-1) - \frac{1}{\pi n^3}4n\pi\cos(2\pi n) = \frac{-4}{n^2}$$

$$b_n = \frac{2}{T}\int_0^T (2nt - t^2)\sin(nt)dt = \frac{1}{\pi}\int (2nt - t^2)\sin(nt)dt$$

$$= \frac{2n}{\pi}\frac{1}{n^2}(\sin(nt) - nt\cos(nt))\Big|_0^{\pi} - \frac{1}{\pi n^3}(2nt\sin(nt) + 2\cos(nt) - n^2t^2\cos(nt))\Big|_0^{2\pi}$$

$$= \frac{-4\pi}{n} + \frac{4\pi}{n} = 0$$

Hence, $$f(t) = \underline{\frac{2\pi^2}{3} - \sum_{n=1}^{\infty}\frac{4}{n^2}\cos(nt)}$$

Chapter 17, Solution 13.

$$T = 2\pi, \ \omega_0 = 1$$

$$a_0 = (1/T)\int_0^T h(t)dt = \frac{1}{2\pi}\left[\int_0^{\pi} 10\sin t\, dt + \int_{\pi}^{2\pi} 20\sin(t-\pi)\, dt\right]$$

$$= \frac{1}{2\pi}\left[-10\cos t\Big|_0^{\pi} - 20\cos(t-\pi)\Big|_{\pi}^{2\pi}\right] = \frac{30}{\pi}$$

$$a_n = (2/T)\int_0^T h(t)\cos(n\omega_0 t)dt$$

$$= [2/(2\pi)]\left[\int_0^{\pi} 10\sin t\cos(nt)dt + \int_{\pi}^{2\pi} 20\sin(t-\pi)\cos(nt)dt\right]$$

Since $\sin A\cos B = 0.5[\sin(A+B) + \sin(A-B)]$
$\sin t\cos nt = 0.5[\sin((n+1)t) + \sin((1-n))t]$
$\sin(t-\pi) = \sin t\cos\pi - \cos t\sin\pi = -\sin t$
$\sin(t-\pi)\cos(nt) = -\sin(t)\cos(nt)$

$$a_n = \frac{1}{2\pi}\left[10\int_0^\pi [\sin([1+n]t) + \sin([1-n]t)]dt - 20\int_\pi^{2\pi}[\sin([1+n]t) + \sin([1-n]t)]dt\right]$$

$$= \frac{5}{\pi}\left[\left(-\frac{\cos([1+n]t)}{1+n} - \frac{\cos([1-n]t)}{1-n}\right)\Bigg|_0^\pi + \left(\frac{2\cos([1+n]t)}{1+n} + \frac{2\cos([1-n]t)}{1-n}\right)\Bigg|_\pi^{2\pi}\right]$$

$$a_n = \frac{5}{\pi}\left[\frac{3}{1+n} + \frac{3}{1-n} - \frac{3\cos([1+n]\pi)}{1+n} - \frac{3\cos([1-n]\pi)}{1-n}\right]$$

But, $\quad [1/(1+n)] + [1/(1-n)] = 1/(1-n^2)$

$\cos([n-1]\pi) = \cos([n+1]\pi) = \cos\pi\cos n\pi - \sin\pi\sin n\pi = -\cos n\pi$

$a_n = (5/\pi)[(6/(1-n^2)) + (6\cos(n\pi)/(1-n^2))]$

$\quad = [30/(\pi(1-n^2))](1 + \cos n\pi) = [-60/(\pi(n-1))], \quad n = \text{even}$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad = 0, \qquad\qquad\qquad n = \text{odd}$

$$b_n = (2/T)\int_0^T h(t)\sin n\omega_o t\, dt$$

$$= [2/(2\pi)][\int_0^\pi 10\sin t\sin nt\, dt + \int_\pi^{2\pi} 20(-\sin t)\sin nt\, dt$$

But, $\quad \sin A\sin B = 0.5[\cos(A-B) - \cos(A+B)]$

$\quad\sin t\sin nt = 0.5[\cos([1-n]t) - \cos([1+n]t)]$

$$b_n = (5/\pi)\{[(\sin([1-n]t)/(1-n)) - (\sin([1+n]t)/(1+n)]\Big|_0^\pi$$

$$+ [(2\sin([1-n]t)/(1-n)) - (2\sin([1+n]t)/(1+n)]\Big|_\pi^{2\pi}\}$$

$$= \frac{5}{\pi}\left[-\frac{\sin([1-n]\pi)}{1-n} + \frac{\sin([1+n]\pi)}{1+n}\right] = 0$$

Thus,
$$h(t) = \frac{30}{\pi} - \frac{60}{\pi}\sum_{k=1}^\infty \frac{\cos(2kt)}{(4k^2 - 1)}$$

Chapter 17, Solution 14.

Since $\cos(A + B) = \cos A \cos B - \sin A \sin B$.

$$f(t) = 2 + \sum_{n=1}^{\infty}\left(\frac{10}{n^3 + 1}\cos(n\pi/4)\cos(2nt) - \frac{10}{n^3 + 1}\sin(n\pi/4)\sin(2nt)\right)$$

Chapter 17, Solution 15.

(a) $D\cos \omega t + E\sin \omega t = A \cos(\omega t - \theta)$

  where $A = \sqrt{D^2 + E^2}$, $\theta = \tan^{-1}(E/D)$

$$A = \sqrt{\frac{16}{(n^2 + 1)^2} + \frac{1}{n^6}}, \quad \theta = \tan^{-1}((n^2+1)/(4n^3))$$

$$f(t) = 10 + \sum_{n=1}^{\infty}\sqrt{\frac{16}{(n^2 + 1)^2} + \frac{1}{n^6}}\cos\left(10nt - \tan^{-1}\frac{n^2 + 1}{4n^3}\right)$$
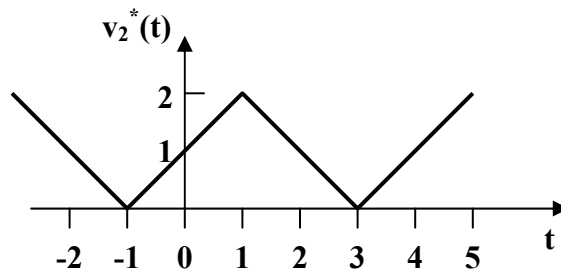
(b) $D\cos \omega t + E\sin \omega t = A \sin(\omega t + \theta)$

  where $A = \sqrt{D^2 + E^2}$, $\theta = \tan^{-1}(D/E)$

$$f(t) = 10 + \sum_{n=1}^{\infty}\sqrt{\frac{16}{(n^2 + 1)^2} + \frac{1}{n^6}}\sin\left(10nt + \tan^{-1}\frac{4n^3}{n^2 + 1}\right)$$

Chapter 17, Solution 16.

If $v_2(t)$ is shifted by 1 along the vertical axis, we obtain $v_2^*(t)$ shown below, i.e.
$v_2^*(t) = v_2(t) + 1$.

Comparing $v_2^*(t)$ with $v_1(t)$ shows that

$$v_2^*(t) \;=\; 2v_1((t + t_o)/2)$$

where $(t + t_o)/2 \;=\; 0$ at $t \;=\; -1$ or $t_o \;=\; 1$

Hence $\qquad\qquad v_2^*(t) \;=\; 2v_1((t + 1)/2)$

But $\qquad\qquad v_2^*(t) \;=\; v_2(t) + 1$

$$v_2(t) + 1 \;=\; 2v_1((t+1)/2)$$

$$v_2(t) \;=\; -1 + 2v_1((t+1)/2)$$

$$= -1 + 1 - \frac{8}{\pi^2}\left[\cos\pi\left(\frac{t+1}{2}\right) + \frac{1}{9}\cos 3\pi\left(\frac{t+1}{2}\right) + \frac{1}{25}\cos 5\pi\left(\frac{t+1}{2}\right) + \cdots\right]$$

$$v_2(t) \;=\; -\frac{8}{\pi^2}\left[\cos\left(\frac{\pi t}{2} + \frac{\pi}{2}\right) + \frac{1}{9}\cos\left(\frac{3\pi t}{2} + \frac{3\pi}{2}\right) + \frac{1}{25}\cos\left(\frac{5\pi t}{2} + \frac{5\pi}{2}\right) + \cdots\right]$$

$$v_2(t) \;=\; -\frac{8}{\pi^2}\left[\sin\left(\frac{\pi t}{2}\right) + \frac{1}{9}\sin\left(\frac{3\pi t}{2}\right) + \frac{1}{25}\sin\left(\frac{5\pi t}{2}\right) + \cdots\right]$$

Chapter 17, Solution 17.

We replace t by –t in each case and see if the function remains unchanged.

(a) $1 - t$, **neither odd nor even.**

(b) $t^2 - 1$, **even**

(c) $\cos n\pi(-t)\sin n\pi(-t) \;=\; -\cos n\pi t\sin n\pi t$, **odd**

(d) $\sin^2 n(-t) \;=\; (-\sin\pi t)^2 \;=\; \sin^2\pi t$, **even**

(e) e^t, **neither odd nor even**.

Chapter 17, Solution 18.

    (a) $T = 2$ leads to $\omega_o = 2\pi/T = \underline{\pi}$

           $f_1(-t) = -f_1(t)$, showing that $f_1(t)$ is **odd and half-wave symmetric**.

    (b) $T = 3$ leads to $\omega_o = 2\pi/3$

           $f_2(t) = f_2(-t)$, showing that $f_2(t)$ is **even**.

    (c) $T = 4$ leads to $\omega_o = \pi/2$

               $f_3(t)$ is **even and half-wave symmetric**.

Chapter 17, Solution 19.

This is a half-wave even symmetric function.

$$a_o = 0 = b_n, \quad \omega_o = 2\pi/T \quad \pi/2$$

$$a_n = \frac{4}{T}\int_0^{T/2}\left[1 - \frac{4t}{T}\right]\cos(n\omega_o t)dt$$

$$= [4/(n\pi)^2](1 - \cos n\pi) \quad = 8/(n^2\pi^2), \quad n = \text{odd}$$
$$= \quad 0, \quad\quad n = \text{even}$$

$$f(t) = \underline{\frac{8}{\pi^2}\sum_{n=odd}^{\infty}\frac{1}{n^2}\cos\left(\frac{n\pi t}{2}\right)}$$

Chapter 17, Solution 20.

This is an even function.

$$b_n = 0, \quad T = 6, \quad \omega = 2\pi/6 = \pi/3$$

$$a_o = \frac{2}{T}\int_0^{T/2}f(t)dt = \frac{2}{6}\left[\int_1^2(4t - 4)dt\int_2^3 4\,dt\right]$$

$$= \frac{1}{3}\left[(2t^2 - 4t)\Big|_1^2 + 4(3-2)\right] = 2$$

$$a_n = \frac{4}{T}\int_0^{T/4} f(t)\cos(n\pi t/3)dt$$

$$= (4/6)\left[\int_1^2 (4t-4)\cos(n\pi t/3)dt + \int_2^3 4\cos(n\pi t/3)dt\right]$$

$$= \frac{16}{6}\left[\frac{9}{n^2\pi^2}\cos\left(\frac{n\pi t}{3}\right) + \frac{3t}{n\pi}\sin\left(\frac{n\pi t}{3}\right) - \frac{3}{n\pi}\sin\left(\frac{n\pi t}{3}\right)\right]_1^2 + \frac{16}{6}\left[\frac{3}{n\pi}\sin\left(\frac{n\pi t}{3}\right)\right]_2^3$$

$$= [24/(n^2\pi^2)][\cos(2n\pi/3) - \cos(n\pi/3)]$$

Thus

$$f(t) = 2 + \frac{24}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\left[\cos\left(\frac{2\pi n}{3}\right) - \cos\left(\frac{\pi n}{3}\right)\right]\cos\left(\frac{n\pi t}{3}\right)$$

At t = 2,

$$f(2) = 2 + (24/\pi^2)[(\cos(2\pi/3) - \cos(\pi/3))\cos(2\pi/3)$$

$$+ (1/4)(\cos(4\pi/3) - \cos(2\pi/3))\cos(4\pi/3)$$

$$+ (1/9)(\cos(2\pi) - \cos(\pi))\cos(2\pi) + \text{-----}]$$

$$= 2 + 2.432(0.5 + 0 + 0.2222 + \text{-----})$$

f(2) = **3.756**

Chapter 17, Solution 21.

This is an even function.

$b_n = 0$, T = 4, $\omega_o = 2\pi/T = \pi/2$.

$$f(t) = 2 - 2t, \qquad 0 < t < 1$$
$$= 0, \qquad\qquad 1 < t < 2$$

$$a_o = \frac{2}{4}\int_0^1 2(1-t)dt = \left[t - \frac{t^2}{2}\right]_0^1 = 0.5$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos(n\omega_o t)dt = \frac{4}{4}\int_0^1 2(1-t)\cos\left(\frac{n\pi t}{2}\right)dt$$

$$= [8/(\pi^2 n^2)][1 - \cos(n\pi/2)]$$

$$f(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{8}{n^2 \pi^2}\left[1 - \cos\left(\frac{n\pi}{2}\right)\right]\cos\left(\frac{n\pi t}{2}\right)$$

Chapter 17, Solution 22.

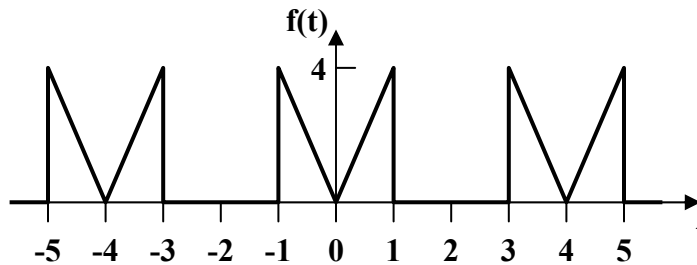Calculate the Fourier coefficients for the function in Fig. 16.54.

f(t)

4

-5 -4 -3 -2 -1 0 1 2 3 4 5   t

Figure 17.61   For Prob. 17.22

This is an even function, therefore $b_n = 0$. In addition, T=4 and $\omega_o = \pi/2$.

$$a_o = \frac{2}{T}\int_0^{T/2} f(t)dt = \frac{2}{4}\int_0^1 4t\,dt = t^2\Big|_0^1 = \underline{\mathbf{1}}$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos(\omega_o nt)dt = \frac{4}{4}\int_0^1 4t\cos(n\pi t/2)dt$$

$$= 4\left[\frac{4}{n^2\pi^2}\cos(n\pi t/2) + \frac{2t}{n\pi}\sin(n\pi t/2)\right]\Bigg|_0^1$$

$$a_n = \frac{16}{n^2\pi^2}(\cos(n\pi/2) - 1) + \frac{8}{n\pi}\sin(n\pi/2)$$

Chapter 17, Solution 23.

f(t) is an odd function.

$$f(t) = t, \quad -1 < t < 1$$

$$a_o = 0 = a_n, \quad T = 2, \quad \omega_o = 2\pi/T = \pi$$

$$b_n = \frac{4}{T} \int_0^{T/2} f(t) \sin(n\omega_0 t)dt = \frac{4}{2} \int_0^1 t \sin(n\pi t)dt$$

$$= \frac{2}{n^2\pi^2} [\sin(n\pi t) - n\pi t \cos(n\pi t)]_0^1$$

$$= -[2/(n\pi)]\cos(n\pi) = 2(-1)^{n+1}/(n\pi)$$

$$f(t) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin(n\pi t)$$

Chapter 17, Solution 24.

(a)   This is an odd function.

$$a_o = 0 = a_n, \quad T = 2\pi, \quad \omega_o = 2\pi/T = 1$$

$$b_n = \frac{4}{T} \int_0^{T/2} f(t) \sin(\omega_o nt)dt$$

$$f(t) = 1 + t/\pi, \qquad 0 < t < \pi$$

$$b_n = \frac{4}{2\pi} \int_0^\pi (1 + t/\pi) \sin(nt)dt$$

$$= \frac{2}{\pi} \left[-\frac{1}{n}\cos(nt) + \frac{1}{n^2\pi}\sin(nt) - \frac{t}{n\pi}\cos(nt) \right]_0^\pi$$

$$= [2/(n\pi)][1 - 2\cos(n\pi)] = [2/(n\pi)][1 + 2(-1)^{n+1}]$$

$$a_2 = \underline{\textbf{0}}, \quad b_2 = [2/(2\pi)][1 + 2(-1)] = -1/\pi = \underline{\textbf{-0.3183}}$$

(b) $\omega n = n\omega_o = 10$ or $n = 10$

$$a_{10} = 0, \quad b_{10} = [2/(10\pi)][1 - \cos(10\pi)] = -1/(5\pi)$$

Thus the magnitude is $A_{10} = \sqrt{a_{10}^2 + b_{10}^2} = 1/(5\pi) = \underline{\textbf{0.06366}}$

and the phase is $\phi_{10} = \tan^{-1}(b_n/a_n) = \underline{\textbf{-90°}}$

(c) $$f(t) = \sum_{n=1}^{\infty} \frac{2}{n\pi}[1 - 2\cos(n\pi)]\sin(nt)\,\pi$$

$$f(\pi/2) = \sum_{n=1}^{\infty} \frac{2}{n\pi}[1 - 2\cos(n\pi)]\sin(n\pi/2)\,\pi$$

For n = 1, $f_1 = (2/\pi)(1 + 2) = 6/\pi$

For n = 2, $f_2 = 0$

For n = 3, $f_3 = [2/(3\pi)][1 - 2\cos(3\pi)]\sin(3\pi/2) = -6/(3\pi)$

For n = 4, $f_4 = 0$

For n = 5, $f_5 = 6/(5\pi)$, ----

Thus, $f(\pi/2) = 6/\pi - 6/(3\pi) + 6/(5\pi) - 6/(7\pi)$ ---------

$$= (6/\pi)[1 - 1/3 + 1/5 - 1/7 + \text{ --------}]$$

$$f(\pi/2) \cong \underline{\mathbf{1.3824}}$$

which is within 8% of the exact value of 1.5.

(d)   From part (c)

$$f(\pi/2) = 1.5 = (6/\pi)[1 - 1/3 + 1/5 - 1/7 + \text{ - - -}]$$

$$(3/2)(\pi/6) = [1 - 1/3 + 1/5 - 1/7 + \text{ - - -}]$$

or $\pi/4 = \underline{\mathbf{1 - 1/3 + 1/5 - 1/7 + \text{ - - -}}}$

Chapter 17, Solution 25.

This is an odd function since $f(-t) = -f(t)$.

$a_o = 0 = a_n$, $T = 3$, $\omega_o = 2\pi/3$.

$$b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin(n\omega_o t)dt = \frac{4}{3}\int_0^1 t\sin(2\pi nt/3)dt$$

$$= \frac{4}{3}\left[\frac{9}{4\pi^2 n^2}\sin\left(\frac{2\pi nt}{3}\right) - \frac{3t}{2n\pi}\cos\left(\frac{2\pi nt}{3}\right)\right]\Bigg|_0^1$$

$$= \frac{4}{3}\left[\frac{9}{4\pi^2 n^2}\sin\left(\frac{2\pi n}{3}\right) - \frac{3}{2n\pi}\cos\left(\frac{2\pi n}{3}\right)\right]$$

$$f(t) = \sum_{n=1}^{\infty}\left[\frac{3}{\pi^2 n^2}\sin\left(\frac{2\pi n}{3}\right) - \frac{2}{n\pi}\cos\left(\frac{2\pi n}{3}\right)\right]\sin\left(\frac{2\pi t}{3}\right)$$

Chapter 17, Solution 26.

$$T = 4, \quad \omega_o = 2\pi/T = \pi/2$$

$$a_o = \frac{1}{T}\int_0^T f(t)dt = \frac{1}{4}\left[\int_0^1 1\,dt + \int_1^3 2\,dt + \int_3^4 1\,dt\right] = 1$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega_o t)dt$$

$$a_n = \frac{2}{4}\left[\int_1^2 1\cos(n\pi t/2)dt + \int_2^3 2\cos(n\pi t/2)dt + \int_3^4 1\cos(n\pi t/2)dt\right]$$

$$= 2\left[\frac{2}{n\pi}\sin\frac{n\pi t}{2}\Bigg|_1^2 + \frac{4}{n\pi}\sin\frac{n\pi t}{2}\Bigg|_2^3 + \frac{2}{n\pi}\sin\frac{n\pi t}{2}\Bigg|_3^4\right]$$

$$= \frac{4}{n\pi}\left[\sin\frac{3n\pi}{2} - \sin\frac{n\pi}{2}\right]$$

$$b_n = \frac{2}{T}\int_0^T f(t)\sin(n\omega_o t)dt$$

$$= \frac{2}{4}\left[\int_1^2 1\sin\frac{n\pi t}{2}\,dt + \int_2^3 2\sin\frac{n\pi t}{2}\,dt + \int_3^4 1\sin\frac{n\pi t}{2}\,dt\right]$$

$$= 2\left[-\frac{2}{n\pi}\cos\frac{n\pi t}{2}\Bigg|_1^2 - \frac{4}{n\pi}\cos\frac{n\pi t}{2}\Bigg|_2^3 - \frac{2}{n\pi}\cos\frac{n\pi t}{2}\Bigg|_3^4\right]$$

$$= \frac{4}{n\pi}\left[\cos(n\pi) - 1\right]$$

Hence

$$f(t) =$$

$$1 + \sum_{n=1}^{\infty} \frac{4}{n\pi}\left[(\sin(3n\pi/2) - \sin(n\pi/2))\cos(n\pi t/2) + (\cos(n\pi) - 1)\sin(n\pi t/2)\right]$$

Chapter 17, Solution 27.

(a) $\qquad$ **odd** symmetry.

(b) $\qquad a_o = 0 = a_n, \ T = 4, \ \omega_o = 2\pi/T = \pi/2$

$$f(t) \quad = t, \quad 0 < t < 1$$

$$= 0, \quad 1 < t < 2$$

$$b_n = \frac{4}{4}\int_0^1 t \sin\frac{n\pi t}{2}\,dt = \left[\frac{4}{n^2\pi^2}\sin\frac{n\pi t}{2} - \frac{2t}{n\pi}\cos\frac{n\pi t}{2}\right]\Big|_0^1$$

$$= \frac{4}{n^2\pi^2}\sin\frac{n\pi}{2} - \frac{2}{n\pi}\cos\frac{n\pi}{2} - 0$$

$$= 4(-1)^{(n-1)/2}/(n^2\pi^2), \qquad n = \text{odd}$$

$$-2(-1)^{n/2}/(n\pi), \qquad n = \text{even}$$

$$a_3 = \underline{\mathbf{0}}, \ b_3 = 4(-1)/(9\pi^2) = \underline{\mathbf{0.045}}$$

(c) $\qquad b_1 = 4/\pi^2, \ b_2 = 1/\pi, \ b_3 = -4/(9\pi^2), \ b_4 = -1/(2\pi), \ b_5 = (25\pi^2)$

$$F_{rms} = \sqrt{a_o^2 + \frac{1}{2}\sum\left(a_n^2 + b_n^2\right)}$$

$$F_{rms}^2 = 0.5\Sigma b_n^2 = [1/(2\pi^2)][(16/\pi^2) + 1 + (16/(8\pi^2)) + (1/4) + (16/(625\pi^2))]$$

$$= (1/19.729)(2.6211 + 0.27 + 0.00259)$$

$$F_{rms} = \sqrt{0.14659} = \underline{\mathbf{0.3829}}$$

Compare this with the exact value of $F_{rms} = \sqrt{\dfrac{2}{T}\int_0^1 t^2 dt} = \sqrt{1/6} = 0.4082$

Chapter 17, Solution 28.

This is half-wave symmetric since $f(t - T/2) = -f(t)$.

$$a_o = 0, \quad T = 2, \quad \omega_o = 2\pi/2 = \pi$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos(n\omega_o t)dt = \frac{4}{2}\int_0^1 (2 - 2t)\cos(n\pi t)dt$$

$$= 4\left[\frac{1}{n\pi}\sin(n\pi t) - \frac{1}{n^2\pi^2}\cos(n\pi t) - \frac{t}{n\pi}\sin(n\pi t)\right]\Bigg|_0^1$$

$$= [4/(n^2\pi^2)][1 - \cos(n\pi)] = \quad \begin{array}{ll} 8/(n^2\pi^2), & n = \text{odd} \\ 0, & n = \text{even} \end{array}$$

$$b_n = 4\int_0^1 (1 - t)\sin(n\pi t)dt$$

$$= 4\left[-\frac{1}{n\pi}\cos(n\pi t) - \frac{1}{n^2\pi^2}\sin(n\pi t) + \frac{t}{n\pi}\cos(n\pi t)\right]\Bigg|_0^1$$

$$= 4/(n\pi), \quad n = \text{odd}$$

$$\underline{f(t) = \sum_{k=1}^{\infty}\left(\frac{8}{n^2\pi^2}\cos(n\pi t) + \frac{4}{n\pi}\sin(n\pi t)\right), \; \mathbf{n = 2k - 1}}$$

Chapter 17, Solution 29.

This function is half-wave symmetric.

$$T = 2\pi, \quad \omega_o = 2\pi/T = 1, \quad f(t) = -t, \quad 0 < t < \pi$$

For odd n, $\quad a_n = \frac{2}{T}\int_0^{\pi} (-t)\cos(nt)dt = -\frac{2}{n^2\pi}[\cos(nt) + nt\sin(nt)]_0^{\pi} = 4/(n^2\pi)$

$$b_n = \frac{2}{\pi}\int_0^{\pi} (-t)\sin(nt)dt = -\frac{2}{n^2\pi}[\sin(nt) - nt\cos(nt)]_0^{\pi} = -2/n$$

Thus,

$$f(t) = 2\sum_{k=1}^{\infty}\left[\frac{2}{n^2\pi}\cos(nt) - \frac{1}{n}\sin(nt)\right], \qquad \underline{n = 2k-1}$$

Chapter 17, Solution 30.

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)e^{-jn\omega_o t}dt = \frac{1}{T}\left[\int_{-T/2}^{T/2} f(t)\cos n\omega_o t\,dt - j\int_{-T/2}^{T/2} f(t)\sin n\omega_o t\,dt\right] \qquad (1)$$

(a)   The second term on the right hand side vanishes if $f(t)$ is even. Hence

$$c_n = \frac{2}{T}\int_0^{T/2} f(t)\cos n\omega_o t\,dt$$

(b)   The first term on the right hand side of (1) vanishes if $f(t)$ is odd. Hence,

$$c_n = -\frac{j2}{T}\int_0^{T/2} f(t)\sin n\omega_o t\,dt$$

Chapter 17, Solution 31.

$$\text{If } h(t) = f(\alpha t), \qquad T' = T/\alpha \qquad \longrightarrow \qquad \omega_o' = \frac{2\pi}{T'} = \frac{2\pi}{T/\alpha} = \underline{\alpha\omega_o}$$

$$a_n' = \frac{2}{T'}\int_0^{T'} h(t)\cos n\omega_o' t\,dt = \frac{2}{T'}\int_0^{T'} f(\alpha t)\cos n\omega_o' t\,dt$$

Let $\alpha t = \lambda,$, $dt = d\lambda/\alpha,$ $\alpha T' = T$

$$a_n' = \frac{2\alpha}{T}\int_0^T f(\lambda)\cos n\omega_o \lambda\,d\lambda/\alpha = a_n$$

Similarly, $\underline{b_n' = b_n}$

Chapter 17, Solution 32.

When $i_s = 1$ (DC component)

$$i = 1/(1 + 2) = 1/3$$

For $n \geq 1$, $\omega_n = 3n$, $I_s = 1/n^2 \angle 0°$

$$I = [1/(1 + 2 + j\omega_n^2)]I_s = I_s/(3 + j6n)$$

$$= \frac{\frac{1}{n^2} \angle 0°}{3\sqrt{1 + 4n^2} \angle \tan^{-1}(6n/3)} = \frac{1}{3n^2\sqrt{1 + 4n^2}} \angle -\tan(2n)$$

Thus,

$$i(t) = \frac{1}{3} + \sum_{n=1}^{\infty} \frac{1}{3n^2\sqrt{1 + 4n^2}} \cos(3n - \tan^{-1}(2n))$$

Chapter 17, Solution 33.

For the DC case, the inductor acts like a short, $V_o = 0$.

For the AC case, we obtain the following:

$$\frac{V_o - V_s}{10} + \frac{V_o}{j2n\pi} + \frac{jn\pi V_o}{4} = 0$$

$$\left(1 + j\left(2.5n\pi - \frac{5}{n\pi}\right)\right)V_o = V_s$$

$$V_o = \frac{V_s}{1 + j\left(2.5n\pi - \frac{5}{n\pi}\right)}$$

$$A_n \angle \Theta_n = \frac{4}{n\pi} \frac{1}{1 + j\left(2.5n\pi - \frac{5}{n\pi}\right)} = \frac{4}{n\pi + j(2.5n^2\pi^2 - 5)}$$
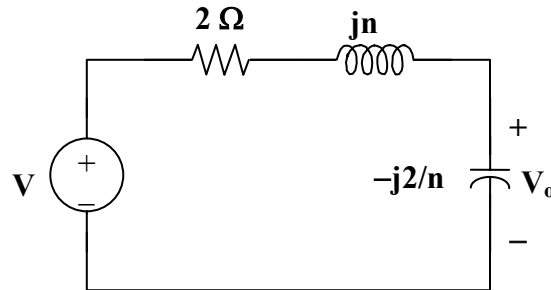
$$A_n = \frac{4}{\sqrt{n^2\pi^2 + (2.5n^2\pi^2 - 5)^2}}; \quad \Theta_n = -\tan^{-1}\left(\frac{2.5n^2\pi^2 - 5}{n\pi}\right)$$

$$v_o(t) = \sum_{n=1}^{\infty} A_n \sin(n\pi t + \Theta_n)\,V$$

Chapter 17, Solution 34.

For any n, $V = [10/n^2]\angle(n\pi/4)$, $\omega = n$.

1 H becomes $j\omega_n L = jn$ and 0.5 F becomes $1/(j\omega_n C) = -j2/n$

$V_o = \{-j(2/n)/[2 + jn - j(2/n)]\}V = \{-j2/[2n + j(n^2 - 2)]\}[(10/n^2)\angle(n\pi/4)]$

$$= \frac{20\angle((n\pi/4) - \pi/2)}{n^2\sqrt{4n^2 + (n^2 - 2)^2}\,\angle\tan^{-1}((n^2 - 2)/2n)}$$

$$= \frac{20}{n^2\sqrt{n^2 + 4}}\,\angle[(n\pi/4) - (\pi/2) - \tan^{-1}((n^2 - 2)/2n)]$$

$$v_o(t) = \sum_{n=1}^{\infty} \frac{20}{n^2\sqrt{n^2 + 4}}\cos\left(nt + \frac{n\pi}{4} - \frac{\pi}{2} - \tan^{-1}\frac{n^2 - 2}{2n}\right)$$

Chapter 17, Solution 35.

If v_s in the circuit of Fig. 17.72 is the same as function $f_2(t)$ in Fig. 17.57(b), determine the dc component and the first three nonzero harmonics of $v_o(t)$.
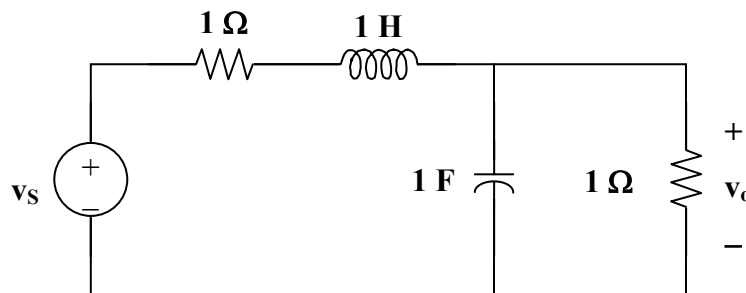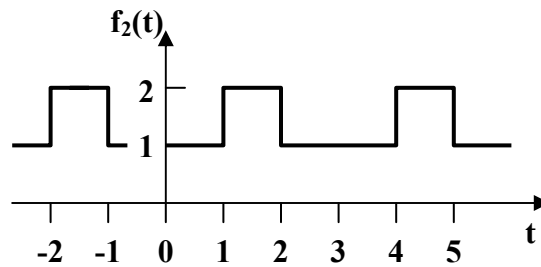

Figure 17.72   For Prob. 17.35

Figure 17.57(b)   For Prob. 17.35

The signal is even, hence, $b_n = 0$. In addition, $T = 3$, $\omega_o = 2\pi/3$.

$$v_s(t) \quad = \quad 1 \;\; \text{for all } 0 < t < 1$$
$$= \quad 2 \;\; \text{for all } 1 < t < 1.5$$

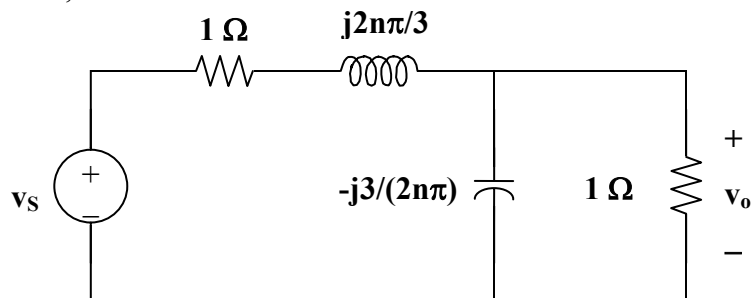$$a_o = \frac{2}{3}\left[\int_0^1 1dt + \int_1^{1.5} 2dt\right] = \frac{4}{3}$$

$$a_n = \frac{4}{3}\left[\int_0^1 \cos(2n\pi t / 3)dt + \int_1^{1.5} 2\cos(2n\pi t / 3)dt\right]$$

$$= \frac{4}{3}\left[\frac{3}{2n\pi}\sin(2n\pi t / 3)\Big|_0^1 + \frac{6}{2n\pi}\sin(2n\pi t / 3)\Big|_1^{1.5}\right] = -\frac{2}{n\pi}\sin(2n\pi / 3)$$

$$v_s(t) = \frac{4}{3} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin(2n\pi / 3)\cos(2n\pi t / 3)$$

Now consider this circuit,

Let $Z = [-j3/(2n\pi)](1)/(1 - j3/(2n\pi)) = -j3/(2n\pi - j3)$

Therefore, $v_o = Zv_s/(Z + 1 + j2n\pi/3)$. Simplifying, we get

$$v_o = \frac{-j9v_s}{12n\pi + j(4n^2\pi^2 - 18)}$$

For the dc case, $n = 0$ and $v_s = \frac{3}{4}$ V and $v_o = v_s/2 = 3/8$ V.

We can now solve for $v_o(t)$

$$v_o(t) = \left[\frac{3}{8} + \sum_{n=1}^{\infty} A_n \cos\left(\frac{2n\pi t}{3} + \Theta_n \right) \right] \text{volts}$$

where $A_n = \dfrac{\dfrac{6}{n\pi} \sin(2n\pi/3)}{\sqrt{16n^2\pi^2 + \left(\dfrac{4n^2\pi^2}{3} - 6 \right)^2}}$ and $\Theta_n = 90^\circ - \tan^{-1}\left(\dfrac{n\pi}{3} - \dfrac{3}{2n\pi} \right)$

where we can further simplify A_n to this, $A_n = \dfrac{9\sin(2n\pi/3)}{n\pi\sqrt{4n^4\pi^4 + 81}}$

Chapter 17, Solution 36.

$$v_s(t) = \sum_{\substack{n=1 \\ n=\text{odd}}}^{\infty} A_n \cos(nt - \theta_n)$$

where $\theta_n = \tan^{-1}[(3/(n\pi))/(-1/(n\pi))] = \tan^{-1}(-3) = 100.5°$

$$A_n = \sqrt{\frac{9}{n^2\pi^2} + \frac{1}{n^2\pi^2} \sin^2 \frac{\pi n}{2}} = \frac{1}{n\pi} \sqrt{9 + \sin^2 \frac{n\pi}{2}}$$

$\omega_n = n$ and 2 H becomes $j\omega_n L = j2n$

Let $\qquad Z = 1\|j2n = j2n/(1 + j2n)$

If V_o is the voltage at the non-reference node or across the 2-H inductor.

$\qquad V_o = ZV_s/(1 + Z) = [j2n/(1 + j2n)]V_s/\{1 + [j2n/(1 + j2n)]\}$

$\qquad = j2nV_s/(1 + j4n)$

But $\qquad V_s = A_n\angle{-\theta_n}$

$\qquad V_o = j2n\, A_n\angle{-\theta_n}/(1 + j4n)$

$$I_o = V_o/j = [2n\, A_n \angle -\theta_n]/\sqrt{1 + 16n^2}\ \angle \tan^{-1} 4n$$

$$= \frac{\dfrac{1}{n\pi}\left(\sqrt{9 + \sin^2 \dfrac{n\pi}{2}}\right)2n}{\sqrt{1 + 16n^2}}\ \angle -100.5° - \tan^{-1} 4n$$

Since $\sin(n\pi/2) = (-1)^{(n-1)/2}$ for n = odd, $\sin^2(n\pi/2) = 1$

$$I_o = \frac{\dfrac{2\sqrt{10}}{\pi}\ \angle -100.5° - \tan^{-1} 4n}{\sqrt{1 + 16n^2}}$$

$$i_o(t) = \frac{2\sqrt{10}}{\pi} \sum_{\substack{n=1 \\ n=odd}}^{\infty} \frac{1}{\sqrt{1+16n^2}} \cos(nt - 100.5° - \tan^{-1} 4n)$$

Chapter 17, Solution 37.

From Example 15.1,

$$v_s(t) = 5 + \frac{20}{\pi} \sum_{k=1}^{\infty} \frac{1}{n} \sin(n\pi t), \qquad n = 2k - 1$$

For the DC component, the capacitor acts like an open circuit.

$$V_o = 5$$

For the nth harmonic,

$$V_s = [20/(n\pi)]\angle 0°$$

10 mF becomes $1/(j\omega_n C) = -j/(n\pi \times 10 \times 10^{-3}) = -j100/(n\pi)$

$$V_o = \frac{-j\dfrac{100}{n\pi}V_s}{-j\dfrac{100}{n\pi} + 20} = \frac{-j100}{20n\pi - j100}\frac{20}{n\pi} = \frac{100\angle - 90° + \tan^{-1}\dfrac{5}{n\pi}}{n\pi\sqrt{25 + n^2\pi^2}}$$

$$v_o(t) = \frac{100}{\pi} \sum \frac{1}{n\sqrt{25 + n^2\pi^2}} \sin(n\pi t - 90° + \tan^{-1}\frac{5}{n\pi})$$

Chapter 17, Solution 38.

$$v_s(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{k=1}^{\infty} \frac{1}{n} \sin n\pi t, \qquad n = 2k+1$$

$$V_o = \frac{j\omega_n}{1 + j\omega_n} V_s, \qquad \omega_n = n\pi$$

For dc, $\omega_n = 0$, $V_s = 0.5$, $V_o = 0$

For nth harmonic, $V_s = \frac{2}{n\pi} \angle -90^o$

$$V_o = \frac{n\pi \angle 90^o}{\sqrt{1 + n^2\pi^2} \angle \tan^{-1} n\pi} \bullet \frac{2}{n\pi} \angle 90^o = \frac{2 \angle -\tan^{-1} n\pi}{\sqrt{1 + n^2\pi^2}}$$

$$v_o(t) = \sum_{k=1}^{\infty} \frac{2}{\sqrt{1 + n^2\pi^2}} \cos(n\pi t - \tan^{-1} n\pi), \qquad n = 2k-1$$

Chapter 17, Solution 39.

Comparing $v_s(t)$ with $f(t)$ in Figure 15.1, v_s is shifted by 2.5 and the magnitude is 5 times that of $f(t)$.
Hence

$$v_s(t) = 5 + \frac{10}{\pi} \sum_{k=1}^{\infty} \frac{1}{n} \sin(n\pi t), \qquad n = 2k - 1$$
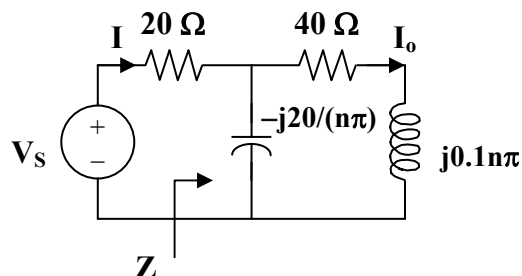
$$T = 2, \quad \omega_o = 2\pi//T = \pi, \quad \omega_n = n\omega_o = n\pi$$

For the DC component, $i_o = 5/(20 + 40) = 1/12$
For the kth harmonic, $V_s = (10/(n\pi)) \angle 0°$

$$100 \text{ mH} \text{ becomes } j\omega_n L = jn\pi \times 0.1 = j0.1n\pi$$
$$50 \text{ mF} \text{ becomes } 1/(j\omega_n C) = -j20/(n\pi)$$

Let $Z = -j20/(n\pi)\|(40 + j0.1n\pi) = \dfrac{-\dfrac{j20}{n\pi}(40 + j0.1n\pi)}{-\dfrac{j20}{n\pi} + 40 + j0.1n\pi}$

$$= \frac{-j20(40 + j0.1n\pi)}{-j20 + 40n\pi + j0.1n^2\pi^2} = \frac{2n\pi - j800}{40n\pi + j(0.1n^2\pi^2 - 20)}$$

$$Z_{in} = 20 + Z = \frac{802n\pi + j(2n^2\pi^2 - 1200)}{40n\pi + j(0.1n^2\pi^2 - 20)}$$

$$I = \frac{V_s}{Z_{in}} = \frac{400n\pi + j(n^2\pi^2 - 200)}{n\pi[802n\pi + j(2n^2\pi^2 - 1200)]}$$

$$I_o = \frac{-\dfrac{j20}{n\pi}I}{-\dfrac{j20}{n\pi} + (40 + j0.1n\pi)} = \frac{-j20I}{40n\pi + j(0.1n^2\pi^2 - 20)}$$

$$= \frac{-j200}{n\pi[802n\pi + j(2n^2\pi^2 - 1200)]}$$

$$= \frac{200\angle -90° - \tan^{-1}\{(2n^2\pi^2 - 1200)/(802n\pi)\}}{n\pi\sqrt{(802)^2 + (2n^2\pi^2 - 1200)^2}}$$

Thus

$$i_o(t) = \underline{\frac{1}{20} + \frac{200}{\pi}\sum_{k=1}^{\infty} I_n \sin(n\pi t - \theta_n),\qquad \mathbf{n = 2k - 1}}$$

where

$$\theta_n = 90° + \tan^{-1}\frac{2n^2\pi^2 - 1200}{802n\pi}$$

$$I_n = \frac{1}{n\sqrt{(804n\pi)^2 + (2n^2\pi^2 - 1200)}}$$

Chapter 17, Solution 40.

$$T = 2, \quad \omega_o = 2\pi/T = \pi$$

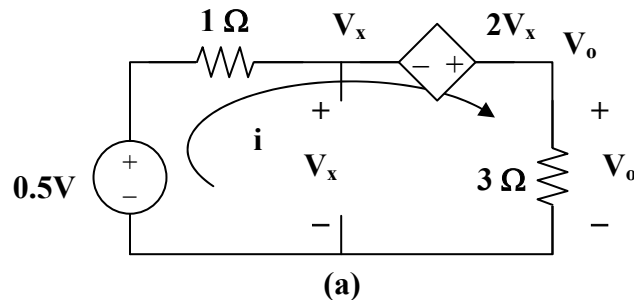$$a_o = \frac{1}{T}\int_0^T v(t)dt = \frac{1}{2}\int_0^1 (2 - 2t)dt = \left[t - \frac{t^2}{2} \right]_0^1 = 1/2$$

$$a_n = \frac{2}{T}\int_0^T v(t) \cos(n\pi t)dt = \int_0^1 2(1 - t) \cos(n\pi t)dt$$

$$= 2\left[\frac{1}{n\pi} \sin(n\pi t) - \frac{1}{n^2\pi^2} \cos(n\pi t) - \frac{t}{n\pi} \sin(n\pi t) \right]_0^1$$

$$= \frac{2}{n^2\pi^2}(1 - \cos n\pi) = \begin{vmatrix} 0, & n = \text{even} \\ \dfrac{4}{n^2\pi^2}, & n = \text{odd} \end{vmatrix} = \frac{4}{\pi^2(2n - 1)^2}$$
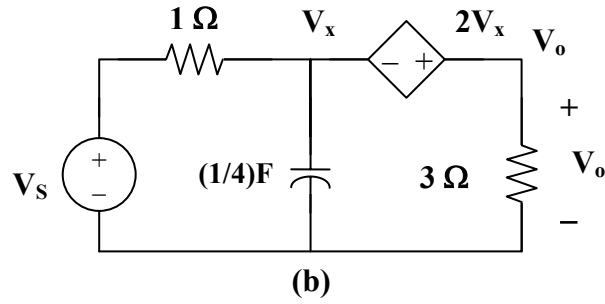
$$b_n = \frac{2}{T}\int_0^T v(t) \sin(n\pi t)dt = 2\int_0^1 (1 - t) \sin(n\pi t)dt$$

$$= 2\left[-\frac{1}{n\pi} \cos(n\pi t) - \frac{1}{n^2\pi^2} \sin(n\pi t) + \frac{t}{n\pi} \cos(n\pi t) \right]_0^1 = \frac{2}{n\pi}$$

$$v_s(t) = \frac{1}{2} + \sum A_n \cos(n\pi t - \varphi_n)$$

$$\text{where} \quad \phi_n = \tan^{-1}\frac{\pi(2n - 1)^2}{2n}, \quad A_n = \sqrt{\frac{4}{n^2\pi^2} + \frac{16}{\pi^4(2n - 1)^4}}$$

For the DC component, $v_s = 1/2$. As shown in Figure (a), the capacitor acts like an open circuit.

(a)

(b)

Applying KVL to the circuit in Figure (a) gives

$$-0.5 - 2V_x + 4i = 0 \tag{1}$$

But
$$-0.5 + i + V_x = 0 \text{ or } -1 + 2V_x + 2i = 0 \tag{2}$$

Adding (1) and (2), $\quad -1.5 + 6i = 0 \text{ or } i = 0.25$

$$V_o = 3i = 0.75$$

For the nth harmonic, we consider the circuit in Figure (b).

$$\omega_n = n\pi, \ V_s = A_n\angle{-\phi}, \ 1/(j\omega_n C) = -j4/(n\pi)$$

At the supernode,
$$(V_s - V_x)/1 = -[n\pi/(j4)]V_x + V_o/3$$

$$V_s = [1 + jn\pi/4]V_x + V_o/3 \tag{3}$$

But $\quad -V_x - 2V_x + V_o = 0 \text{ or } V_o = 3V_x$

Substituting this into (3),
$$V_s = [1 + jn\pi/4]V_x + V_x = [2 + jn\pi/4]V_x$$

$$= (1/3)[2 + jn\pi/4]V_o = (1/12)[8 + jn\pi]V_o$$

$$V_o = 12V_s/(8 + jn\pi) = \frac{12A_n\angle{-\phi}}{\sqrt{64 + n^2\pi^2}\angle{\tan^{-1}(n\pi/8)}}$$

$$V_o = \frac{12}{\sqrt{64 + n^2\pi^2}}\sqrt{\frac{4}{n^2\pi^2} + \frac{16}{\pi^4(2n-1)^4}}\angle[\tan^{-1}(n\pi/8) - \tan^{-1}(\pi(2n-1)/(2n))]$$

Thus

$$v_o(t) = \frac{3}{4} + \sum_{n=1}^{\infty} V_n \cos(n\pi t + \theta_n)$$

where

$$V_n = \frac{12}{\sqrt{64 + n^2\pi^2}} \sqrt{\frac{4}{n^2\pi^2} + \frac{16}{\pi^4(2n-1)^4}}$$

$$\theta_n = \tan^{-1}(n\pi/8) - \tan^{-1}(\pi(2n-1)/(2n))$$

Chapter 17, Solution 41.

For the full wave rectifier,

$$T = \pi, \quad \omega_o = 2\pi/T = 2, \quad \omega_n = n\omega_o = 2n$$

Hence

$$v_{in}(t) = \frac{2}{\pi} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{4n^2 - 1} \cos(2nt)$$

For the DC component,

$$V_{in} = 2/\pi$$

The inductor acts like a short-circuit, while the capacitor acts like an open circuit.
$$V_o = V_{in} = 2/\pi$$

For the nth harmonic,
$$V_{in} = [-4/(\pi(4n^2 - 1))]\angle 0°$$

$$2\ H\ \text{becomes}\ j\omega_n L = j4n$$

$$0.1\ F\ \text{becomes}\ 1/(j\omega_n C) = -j5/n$$

$$Z = 10\|(-j5/n) = -j10/(2n - j)$$

$$V_o = [Z/(Z + j4n)]V_{in} = -j10V_{in}/(4 + j(8n - 10))$$

$$= -\frac{j10}{4 + j(8n - 10)}\left(-\frac{4\angle 0°}{\pi(4n^2 - 1)}\right)$$

$$= \frac{40\angle\{90° - \tan^{-1}(2n - 2.5)\}}{\pi(4n^2 - 1)\sqrt{16 + (8n - 10)^2}}$$

Hence $\qquad v_o(t) = \frac{2}{\pi} + \sum_{n=1}^{\infty} A_n \cos(2nt + \theta_n)$

where

$$A_n = \frac{20}{\pi(4n^2 - 1)\sqrt{16n^2 - 40n + 29}}$$

$$\theta_n = \underline{90° - \tan^{-1}(2n - 2.5)}$$

Chapter 17, Solution 42.

$$v_s = 5 + \frac{20}{\pi} \sum_{k=1}^{\infty} \frac{1}{n} \sin n\pi t, \quad n = 2k - 1$$

$$\frac{V_s - 0}{R} = j\omega_n C(0 - V_o) \qquad \longrightarrow \qquad V_o = \frac{j}{\omega_n RC} V_s, \quad \omega_n = n\omega_o = n\pi$$

For $n = 0$ (dc component), $V_o = 0$.
For the nth harmonic,

$$V_o = \frac{1\angle 90°}{n\pi RC} \frac{20}{n\pi} \angle - 90° = \frac{20}{n^2\pi^2 \times 10^4 \times 40 \times 10^{-9}} = \frac{10^5}{2n^2\pi^2}$$

Hence,

$$v_o(t) = \frac{10^5}{2\pi^2} \sum_{k=1}^{\infty} \frac{1}{n^2} \cos n\pi t, \quad n = 2k - 1$$

Alternatively, we notice that this is an integrator so that

$$v_o(t) = -\frac{1}{RC} \int v_s dt = \frac{10^5}{2\pi^2} \sum_{k=1}^{\infty} \frac{1}{n^2} \cos n\pi t, \quad n = 2k - 1$$

Chapter 17, Solution 43.

(a) $\quad V_{rms} = \sqrt{a_0^2 + \dfrac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)} = \sqrt{30^2 + \dfrac{1}{2}(20^2 + 10^2)} = \underline{\mathbf{33.91\ V}}$

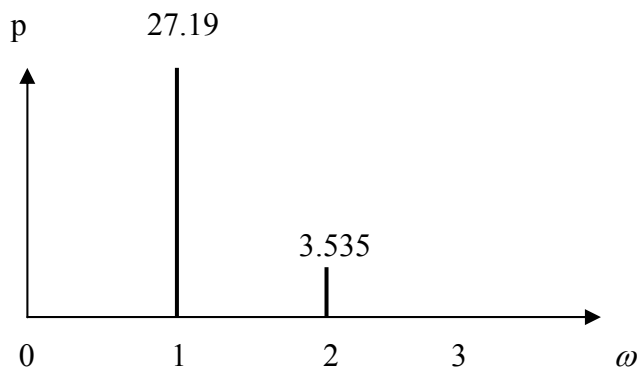(b) $\quad I_{rms} = \sqrt{6^2 + \dfrac{1}{2}(4^2 + 2^2)} = \underline{\mathbf{6.782\ A}}$

(c) $\quad P = V_{dc}I_{dc} + \dfrac{1}{2}\sum V_n I_n \cos(\Theta_n - \Phi_n)$

$\qquad = 30\text{x}6 + 0.5[20\text{x}4\cos(45^o\text{-}10^o) - 10\text{x}2\cos(\text{-}45^o\text{+}60^o)]$

$\qquad = 180 + 32.76 - 9.659 = \underline{\mathbf{203.1\ W}}$

Chapter 17, Solution 44.

(a) $\quad p = vi = \dfrac{1}{2}\left[60\cos 25^o + 10\cos 45^o + 0\right] = 27.19 + 3.535 + 0 = \underline{30.73\ W}$

(b) The power spectrum is shown below.

Chapter 17, Solution 45.

$\omega_n = 1000n$

$\qquad j\omega_n L = j1000n\text{x}2\text{x}10^{-3} = j2n$

$\qquad 1/(j\omega_n C) = -j/(1000n\text{x}40\text{x}10^{-6}) = -j25/n$

$$Z = R + j\omega_n L + 1/(j\omega_n C) = 10 + j2n - j25/n$$

$$I = V/Z$$

For n = 1, $V_1 = 100$, $Z = 10 + j2 - j25 = 10 - j23$

$$I_1 = 100/(10 - j23) = 3.987\angle 73.89°$$

For n = 2, $V_2 = 50$, $Z = 10 + j4 - j12.5 = 10 - j8.5$

$$I_2 = 50/(10 - j8.5) = 3.81\angle 40.36°$$

For n = 3, $V_3 = 25$, $Z = 10 + j6 - j25/3 = 10 - j2.333$

$$I_3 = 25/(10 - j2.333) = 2.435\angle 13.13°$$

$$I_{rms} = 0.5\sqrt{3.987^2 + 3.81^2 + 2.435^2} = \underline{\textbf{3.014 A}}$$

$$p = V_{DC}I_{DC} + \frac{1}{2}\sum_{n=1}^{3} V_n I_n \cos(\theta_n - \phi_n)$$

$$= 0 + 0.5[100x3.987\cos(73.89°) + 50x3.81\cos(40.36°) + 25x2.435\cos(13.13°)]$$

$$= 0.5[110.632 + 145.16 + 59.28] = \underline{\textbf{157.54 watts}}$$

Chapter 17, Solution 46.

(a)   This is an even function

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T f^2(t)dt} = \sqrt{\frac{2}{T}\int_0^{T/2} f^2(t)dt}$$

$$f(t) = \begin{vmatrix} 2 - 2t, & 0 < t < 1 \\ 0, & 1 < t < 2 \end{vmatrix}$$

T = 4, $\omega_o = 2\pi/T = \pi/2$

$$I_{rms}^2 = \frac{2}{4}\int_0^1 4(1-t)^2 dt = 2(t - t^2 + t^3/3)\Big|_0^1$$

$$= 2(1 - 1 + 1/3) = 2/3 \text{ or}$$

$$I_{rms} = \underline{\mathbf{0.8165\ A}}$$

(b)   From Problem 16.14,

$$a_n = [8/(n^2\pi^2)][1 - \cos(n\pi/2)], \quad a_o = 0.5$$

$$a_1 = 8/\pi^2, \quad a_2 = 4/\pi^2, \quad a_3 = 8/(9\pi^2), \quad a_4 = 0, \quad a_5 = 9/(25\pi^2), \quad a_6 = 4/(9\pi^2)$$

$$I_{rms} = \sqrt{a_o + \frac{1}{2}\sum_{n=1}^{\infty} A_n^2} \cong \sqrt{\frac{1}{4} + \frac{1}{2\pi^4}\left(64 + 16 + \frac{64}{81} + \frac{64}{625} + \frac{16}{81}\right)} = \sqrt{0.66623}$$

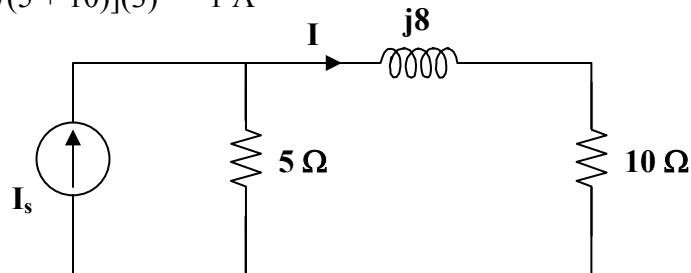$$I_{rms} = \underline{\mathbf{0.8162\ A}}$$

Chapter 17, Solution 47.

Let $I = I_{DC} + I_1 + I_2$

For the DC component

$$I_{DC} = [5/(5 + 10)](3) = 1\ A$$

For AC, $\omega = 100$

$$j\omega L = j100 \times 80 \times 10^{-3} = j8$$

$$I_n = 5I_s/(5 + 10 + j8)$$

For $I_s = 0.5\angle{-60}°$

$$I_1 = 10\angle{-60}°/(15 + j8) \text{ or } |I_1| = 10/\sqrt{15^2 + 8^2}$$

For $I_s = 0.5\angle{-120}°$

$$I_2 = 2.5\angle{-120}°/(15 + j8) \text{ or } |I_2| = 2.5/\sqrt{15^2 + 8^2}$$

$$p_{10} = (I_{DC}^2 + |I_1|^2/2 + |I_2|^2/2)10 = (1 + [100/(2 \times 289)] + [6.25/(2 \times 289)]) \times 10$$

$$p_{10} = \underline{\mathbf{11.838\ watts}}$$

Chapter 17, Solution 48.

(a) For the DC component, $i(t) = 20$ mA. The capacitor acts like an open circuit so that
$$v = Ri(t) = 2 \times 10^3 \times 20 \times 10^{-3} = 40$$

For the AC component,
$$\omega_n = 10n, \quad n = 1,2$$

$$1/(j\omega_n C) = -j/(10n \times 100 \times 10^{-6}) = (-j/n) \text{ k}\Omega$$

$$Z = 2\|(-j/n) = 2(-j/n)/(2 - j/n) = -j2/(2n - j)$$

$$V = ZI = [-j2/(2n - j)]I$$

For n = 1, $V_1 = [-j2/(2 - j)]16\angle 45° = 14.311\angle -18.43°$ mV

For n = 2, $V_2 = [-j2/(4 - j)]12\angle -60° = 5.821\angle -135.96°$ mV

$v(t) = \underline{40 + 0.014311\cos(10t - 18.43°) + 0.005821\cos(20t - 135.96°) \text{ V}}$

(b) $p = V_{DC}I_{DC} + \dfrac{1}{2}\displaystyle\sum_{n=1}^{\infty} V_n I_n \cos(\theta_n - \phi_n)$

$$= 20 \times 40 + 0.5 \times 10 \times 0.014311\cos(45° + 18.43°)$$
$$+ 0.5 \times 12 \times 0.005821\cos(-60° + 135.96°)$$

$$= \underline{\textbf{800.1 mW}}$$

Chapter 17, Solution 49.

(a) $Z^2{}_{rms} = \dfrac{1}{T}\displaystyle\int_0^T z^2(t)dt = \dfrac{1}{2\pi}\left[\displaystyle\int_0^\pi 1dt + \displaystyle\int_\pi^{2\pi} 4dt\right] = \dfrac{1}{2\pi}(5\pi) = 2.5$

$$\underline{Z_{rms} = 1.581}$$

(b)
$Z^2{}_{rms} = a^2{}_o + \dfrac{1}{2}\displaystyle\sum_{n=1}^{\infty}(a^2{}_n + b^2{}_n) = \dfrac{1}{4} + \dfrac{1}{2}\displaystyle\sum_{n=1}^{\infty}\dfrac{36}{n^2\pi^2} = \dfrac{1}{4} + \dfrac{1}{18\pi^2}\left(1 + \dfrac{1}{4} + \dfrac{1}{9} + \dfrac{1}{16} + \dfrac{1}{25} + ...\right) = 2.9193$

$$\underline{Z_{rms} = 1.7086}$$

(c) $\%error = \left(\dfrac{1.7086}{1.581} - 1\right) \times 100 = \underline{8.071}$

Chapter 17, Solution 50.

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-j\omega_o nt}\,dt, \qquad \omega_o = \frac{2n}{1} = \pi$$

$$= \frac{1}{2}\int_-^1 te^{-jn\pi t}\,dt$$

Using integration by parts,

$$u = t \text{ and } du = dt$$
$$dv = e^{-jn\pi t}dt \text{ which leads to } v = -[1/(2jn\pi)]e^{-jn\pi t}$$

$$c_n = -\left.\frac{t}{2jn\pi}e^{-jn\pi t}\right|_{-1}^1 + \frac{1}{2jn\pi}\int_{-1}^1 e^{-jn\pi t}\,dt$$

$$= \frac{j}{n\pi}\left[e^{-jn\pi} + e^{jn\pi t}\right] + \left.\frac{1}{2n^2\pi^2(-j)^2}e^{-jn\pi t}\right|_{-1}^1$$

$$= [j/(n\pi)]\cos(n\pi) + [1/(2n^2\pi^2)](e^{-jn\pi} - e^{jn\pi})$$

$$c_n = \frac{j(-1)^n}{n\pi} + \frac{2j}{2n^2\pi^2}\sin(n\pi) = \frac{j(-1)^n}{n\pi}$$

Thus

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_o t} = \underline{\sum_{n=-\infty}^{\infty}(-1)^n \frac{j}{n\pi}e^{jn\pi t}}$$

Chapter 17, Solution 51.

$$T = 2, \qquad \omega_0 = 2\pi/T = \pi$$

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-jn\omega_o t}\,dt = \frac{1}{2}\int_0^2 t^2 e^{-jn\pi t}\,dt = \frac{1}{2}\frac{e^{-jn\pi t}}{(-jn\pi)^3}\left(-n^2\pi^2 t^2 + 2jn\pi t + 2\right)\Big|_0^2$$

$$c_n = \frac{1}{j2n^3\pi^3}(-4n^2\pi^2 + j4n\pi) = \frac{2}{n^2\pi^2}(1 + jn\pi)$$

$$f(t) = \underline{\sum_{n=-\infty}^{\infty} \frac{2}{n^2\pi^2}(1 + jn\pi)e^{jn\pi t}}$$

Chapter 17, Solution 52.

$$c_n = \frac{1}{T}\int_0^T f(t)e^{-j\omega_o nt}dt, \quad \omega_o = \frac{2n}{1} = \pi$$

$$= \frac{1}{2}\int_{-}^{1} te^{-jn\pi t}dt$$

Using integration by parts,

$$u = t \text{ and } du = dt$$
$$dv = e^{-jn\pi t}dt \text{ which leads to } v = -[1/(2jn\pi)]e^{-jn\pi t}$$

$$c_n = -\frac{t}{2jn\pi}e^{-jn\pi t}\bigg|_{-1}^{1} + \frac{1}{2jn\pi}\int_{-1}^{1}e^{-jn\pi t}dt$$

$$= \frac{j}{n\pi}\left[e^{-jn\pi} + e^{jn\pi t}\right] + \frac{1}{2n^2\pi^2(-j)^2}e^{-jn\pi t}\bigg|_{-1}^{1}$$

$$= [j/(n\pi)]\cos(n\pi) + [1/(2n^2\pi^2)](e^{-jn\pi} - e^{jn\pi})$$

$$c_n = \frac{j(-1)^n}{n\pi} + \frac{2j}{2n^2\pi^2}\sin(n\pi) = \frac{j(-1)^n}{n\pi}$$

Thus

$$f(t) = \sum_{n=-\infty}^{\infty}c_n e^{jn\omega_o t} = \underline{\sum_{n=-\infty}^{\infty}(-1)^n\frac{j}{n\pi}e^{jn\pi t}}$$

Chapter 17, Solution 53.

$$\omega_o = 2\pi/T = 2\pi$$

$$c_n = \int_0^T e^{-t}e^{-jn\omega_o t}dt = \int_0^1 e^{-(1+jn\omega_o)t}dt$$

$$= \frac{-1}{1+j2\pi n}e^{-(1+j2n\pi)t}\bigg|_0^1 = \frac{-1}{1+j2n\pi}\left[e^{-(1+j2n\pi)} - 1\right]$$

$$= [1/(j2n\pi)][1 - e^{-1}(\cos(2\pi n) - j\sin(2n\pi))]$$
$$= (1 - e^{-1})/(1 + j2n\pi) = 0.6321/(1 + j2n\pi)$$

$$f(t) = \underline{\sum_{n=-\infty}^{\infty}\frac{0.6321e^{j2n\pi t}}{1 + j2n\pi}}$$

Chapter 17, Solution 54.

$T = 4$, $\omega_0 = 2\pi/T = \pi/2$

$$c_n = \frac{1}{T} \int_0^T f(t)e^{-j\omega_0 nt}dt$$

$$= \frac{1}{4}\left[\int_0^1 2e^{-jn\pi t/2}dt + \int_1^2 1e^{-jn\pi t/2}dt - \int_2^4 1e^{-jn\pi t/2}dt\right]$$

$$= \frac{j}{2n\pi}\left[2e^{-jn\pi/2} - 2 + e^{-jn\pi} - e^{-jn\pi/2} - e^{-j2n\pi} + e^{-jn\pi}\right]$$

$$= \frac{j}{2n\pi}\left[3e^{-jn\pi/2} - 3 + 2e^{-jn\pi}\right]$$

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

Chapter 17, Solution 55.

$T = 2\pi$, $\omega_0 = 2\pi/T = 1$

$$c_n = \frac{1}{T} \int_0^T i(t)e^{-jn\omega_0 t}dt$$

But

$$i(t) = \begin{vmatrix} \sin(t), & 0 < t < \pi \\ 0, & \pi < t < 2\pi \end{vmatrix}$$

$$c_n = \frac{1}{2\pi} \int_0^\pi \sin(t)e^{-jn\pi t}dt = \frac{1}{2\pi} \int_0^\pi \frac{1}{2j}(e^{jt} - e^{-jt})e^{-jnt}dt$$

$$= \frac{1}{4\pi j}\left[\frac{e^{jt(1-n)}}{j(1-n)} + \frac{e^{-jt(1+n)}}{j(1+n)}\right]_0^\pi$$

$$= -\frac{1}{4}\left[\frac{e^{j\pi(1-n)} - 1}{1-n} + \frac{e^{-j\pi(n+1)} - 1}{1+n}\right]$$

$$= \frac{1}{4\pi(n^2 - 1)} \left[e^{j\pi(1-n)} - 1 + ne^{j\pi(1-n)} - n + e^{-j\pi(1+n)} - 1 - ne^{-j\pi(1+n)} + n \right]$$

But $e^{j\pi} = \cos(\pi) + j\sin(\pi) = -1 = e^{-j\pi}$

$$c_n = \frac{1}{4\pi(n^2 - 1)} \left[-e^{-jn\pi} - e^{-jn\pi} - ne^{-jn\pi} + ne^{-jn\pi} - 2 \right] = \frac{1 + e^{-jn\pi}}{2\pi(1 - n^2)}$$

Thus

$$i(t) = \sum_{n=-\infty}^{\infty} \frac{1 + e^{-jn\pi}}{2\pi(1 - n^2)} e^{jn\pi t}$$

Chapter 17, Solution 56.

$c_o = a_o = 10, \ \omega_o = \pi$

$c_o = (a_n - jb_n)/2 = (1 - jn)/[2(n^2 + 1)]$

$$f(t) = 10 + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \frac{(1 - jn)}{2(n^2 + 1)} e^{jn\pi t}$$

Chapter 17, Solution 57.

$a_o = (6/-2) = -3 = c_o$

$c_n = 0.5(a_n - jb_n) = a_n/2 = 3/(n^3 - 2)$

$$f(t) = -3 + \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} \frac{3}{n^3 - 2} e^{j50nt}$$

Chapter 17, Solution 58.

$c_n = (a_n - jb_n)/2, \ (-1) = \cos(n\pi), \ \omega_o = 2\pi/T = 1$

$c_n = [(\cos(n\pi) - 1)/(2\pi n^2)] - j\cos(n\pi)/(2n)$

Thus

$$f(t) = \frac{\pi}{4} + \sum \left(\frac{\cos(n\pi) - 1}{2\pi n^2} - j \frac{\cos(n\pi)}{2n} \right) e^{jnt}$$

Chapter 17, Solution 59.

For f(t), $T = 2\pi$, $\omega_o = 2\pi/T = 1$.
$\qquad a_o = $ DC component $= (1 \times \pi + 0)/2\pi = 0.5$

For $\quad$ h(t), $T = 2$, $\omega_o = 2\pi/T = \pi$.
$\qquad a_o = (3 \times 1 - 2 \times 1)/2 = 0.5$

Thus by replacing $\omega_o = 1$ with $\omega_o = \pi$ and multiplying the magnitude by five, we obtain

$$h(t) = \frac{1}{2} - \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{j5e^{j(2n+1)\pi t}}{(2n+1)\pi}$$

Chapter 17, Solution 60.

From Problem 16.17,

$\qquad a_o = 0 = a_n$, $\quad b_n = [2/(n\pi)][1 - 2\cos(n\pi)]$, $\quad c_o = \underline{\mathbf{0}}$

$\qquad c_n = (a_n - jb_n)/2 = \underline{\mathbf{[j/(n\pi)][2\cos(n\pi) - 1], \ n \neq 0}}$.

Chapter 17, Solution 61.

(a) $\qquad\qquad\qquad\qquad\qquad \omega_o = 1$.

$$f(t) = a_o + \sum A_n \cos(n\omega_o t - \phi_n)$$

$$= 6 + 4\cos(t + 50°) + 2\cos(2t + 35°)$$
$$+ \cos(3t + 25°) + 0.5\cos(4t + 20°)$$

$$= 6 + 4\cos(t)\cos(50°) - 4\sin(t)\sin(50°) + 2\cos(2t)\cos(35°)$$
$$- 2\sin(2t)\sin(35°) + \cos(3t)\cos(25°) - \sin(3t)\sin(25°)$$
$$+ 0.5\cos(4t)\cos(20°) - 0.5\sin(4t)\sin(20°)$$

$$= \underline{\mathbf{6 + 2.571\cos(t) - 3.73\sin(t) + 1.635\cos(2t)}}$$
$$\underline{\mathbf{- 1.147\sin(2t) + 0.906\cos(3t) - 0.423\sin(3t)}}$$
$$\underline{\mathbf{+ 0.47\cos(4t) - 0.171\sin(4t)}}$$

(b) $$f_{rms} = \sqrt{a_o^2 + \frac{1}{2}\sum_{n=1}^{\infty} A_n^2}$$

$$f_{rms}^2 = 6^2 + 0.5[4^2 + 2^2 + 1^2 + (0.5)^2] = 46.625$$

$$f_{rms} = \underline{\textbf{6.828}}$$

Chapter 17, Solution 62.

(a) $\omega_o = 20 = 2\pi / T \qquad \longrightarrow \qquad T = \dfrac{2\pi}{20} = \underline{0.3141s}$

(b) $f(t) = a_o + \displaystyle\sum_{n=1}^{\infty} A_n \cos(n\omega_o t + \phi_n) = 3 + 4\cos(20t + 90^o) + 5.1\cos(40t + 90^o) + ...$

$\underline{f(t) = 3 - 4\sin 20t - 5.1\sin 40t - 2.7\sin 60t - 1.8\sin 80t - ....}$

Chapter 17, Solution 63.

This is an even function.

$T = 3$, $\omega_o = 2\pi/3$, $b_n = 0$.

$$f(t) = \begin{vmatrix} 1, & 0 < t < 1 \\ 2, & 1 < t < 1.5 \end{vmatrix}$$

$$a_o = \frac{2}{T}\int_0^{T/2} f(t)dt = \frac{2}{3}\left[\int_0^1 1dt + \int_1^{1.5} 2dt\right] = (2/3)[1 + 1] = 4/3$$

$$a_n = \frac{4}{T}\int_0^{T/2} f(t)\cos(n\omega_o t)dt = \frac{4}{3}\left[\int_0^1 1\cos(2n\pi t/3)dt + \int_1^{1.5} 2\cos(2n\pi t/3)dt\right]$$

$$= \frac{4}{3}\left[\frac{3}{2n\pi}\sin\left(\frac{2n\pi t}{3}\right)\Big|_0^1 + \frac{6}{2n\pi}\sin\left(\frac{2n\pi t}{3}\right)\Big|_1^{1.5}\right]$$
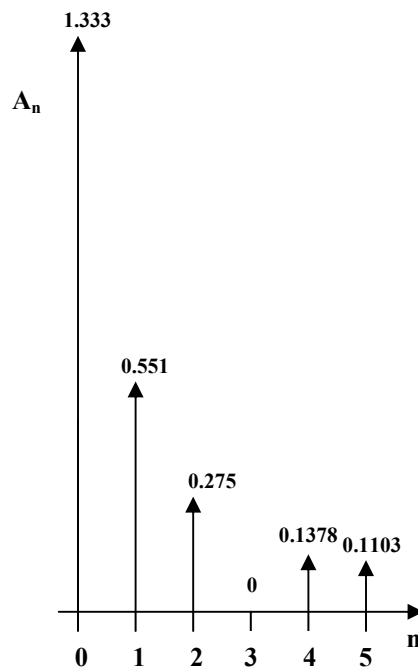
$$= [-2/(n\pi)]\sin(2n\pi/3)$$

$$f_2(t) = \frac{4}{3} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{3n\pi}{3}\right)\cos\left(\frac{2n\pi t}{3}\right)$$

$a_o = 4/3 = 1.3333$, $\omega_o = 2\pi/3$, $a_n = -[2/(n\pi)]\sin(2n\pi t/3)$

$$A_n = \sqrt{a_n^2 + b_n^2} = \left|\frac{2}{n\pi}\sin\left(\frac{2n\pi}{3}\right)\right|$$

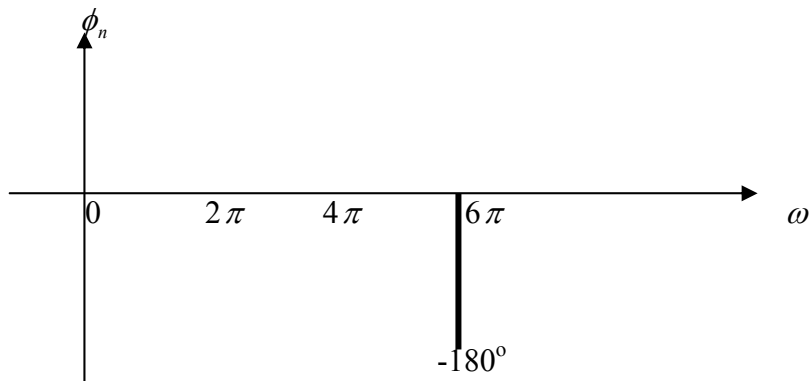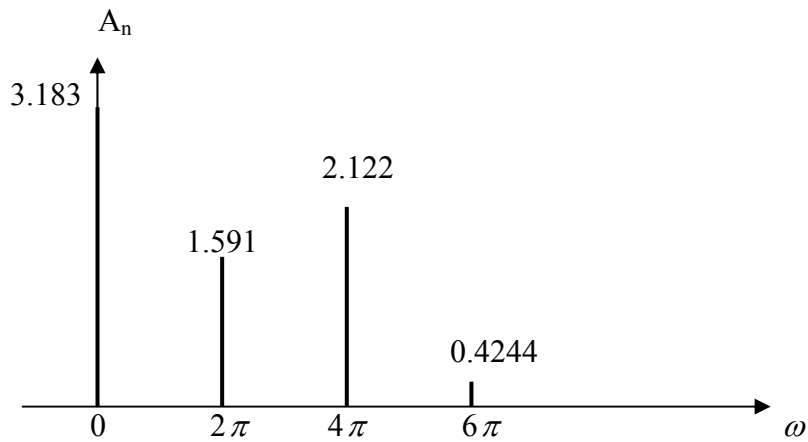$A_1 = 0.5513$, $A_2 = 0.2757$, $A_3 = 0$, $A_4 = 0.1375$, $A_5 = 0.1103$

The amplitude spectra are shown below.

Chapter 17, Solution 64.

The amplitude and phase spectra are shown below.

Chapter 17, Solution 65.

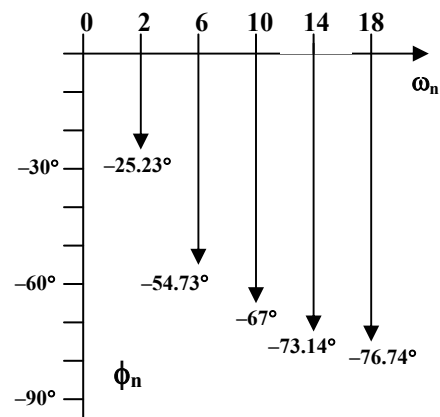$$a_n = 20/(n^2\pi^2), \quad b_n = -3/(n\pi), \quad \omega_n = 2n$$
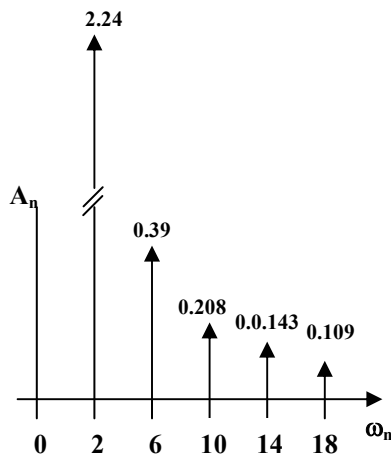
$$A_n = \sqrt{a_n^2 + b_n^2} = \sqrt{\frac{400}{n^4\pi^4} + \frac{9}{n^2\pi^2}}$$

$$= \frac{3}{n\pi}\sqrt{1 + \frac{44.44}{n^2\pi^2}}, \quad n = 1,\ 3,\ 5,\ 7,\ 9,\ \text{etc.}$$

n	A_n
1	2.24
3	0.39
5	0.208
7	0.143
9	0.109

$$\phi_n = \tan^{-1}(b_n/a_n) = \tan^{-1}\{[-3/(n\pi)][n^2\pi^2/20]\} = \tan^{-1}(-nx0.4712)$$

n	ϕ_n
1	−25.23°
3	−54.73°
5	−67°
7	−73.14°
9	−76.74°
∞	−90°

Chapter 17, Solution 66.

The schematic is shown below. The waveform is inputted using the attributes of VPULSE. In the Transient dialog box, we enter Print Step = 0.05, Final Time = 12, Center Frequency = 0.5, Output Vars = V(1) and click enable Fourier. After simulation, the output plot is shown below. The output file includes the following Fourier components.
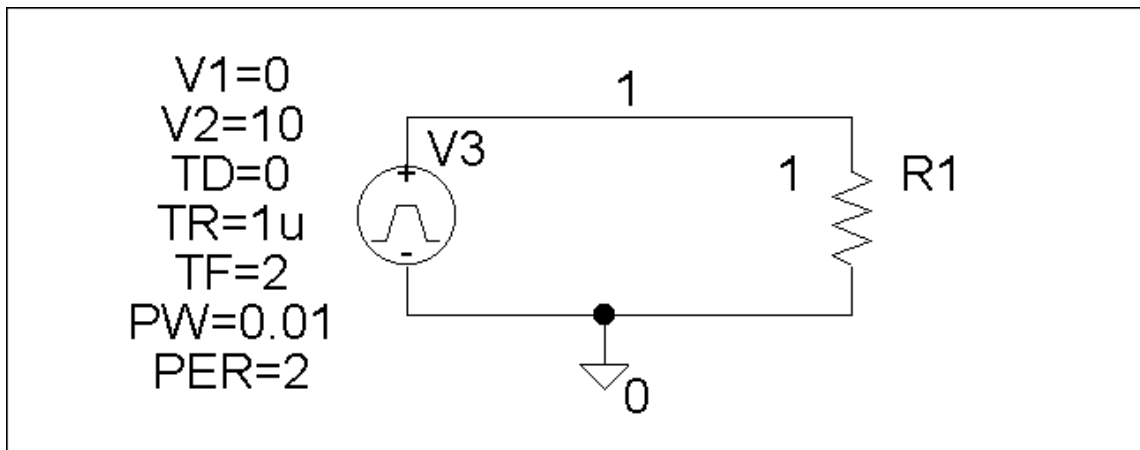
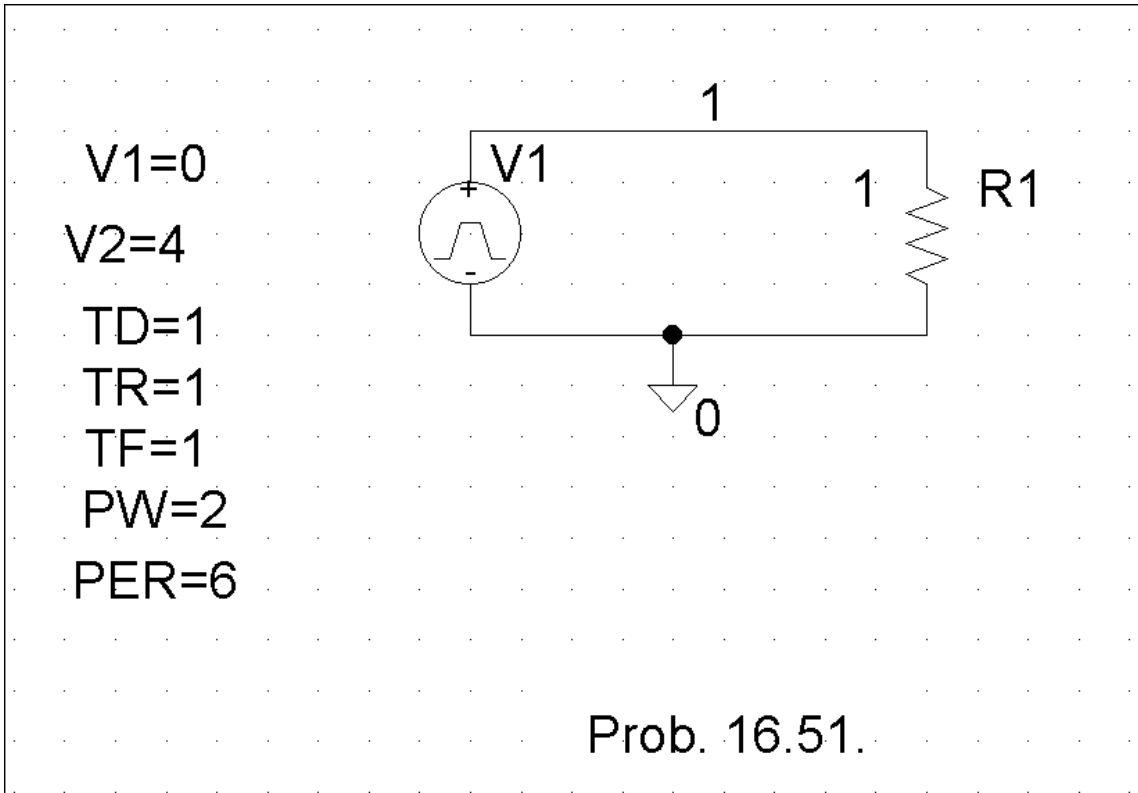FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

DC COMPONENT = 5.099510E+00

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	5.000E-01	3.184E+00	1.000E+00	1.782E+00	0.000E+00
2	1.000E+00	1.593E+00	5.002E-01	3.564E+00	1.782E+00
3	1.500E+00	1.063E+00	3.338E-01	5.347E+00	3.564E+00
4	2.000E+00	7.978E-01	2.506E-01	7.129E+00	5.347E+00
5	2.500E+00	6.392E-01	2.008E-01	8.911E+00	7.129E+00
6	3.000E+00	5.336E-01	1.676E-01	1.069E+01	8.911E+00
7	3.500E+00	4.583E-01	1.440E-01	1.248E+01	1.069E+01
8	4.000E+00	4.020E-01	1.263E-01	1.426E+01	1.248E+01
9	4.500E+00	3.583E-01	1.126E-01	1.604E+01	1.426E+01

TOTAL HARMONIC DISTORTION = 7.363360E+01 PERCENT
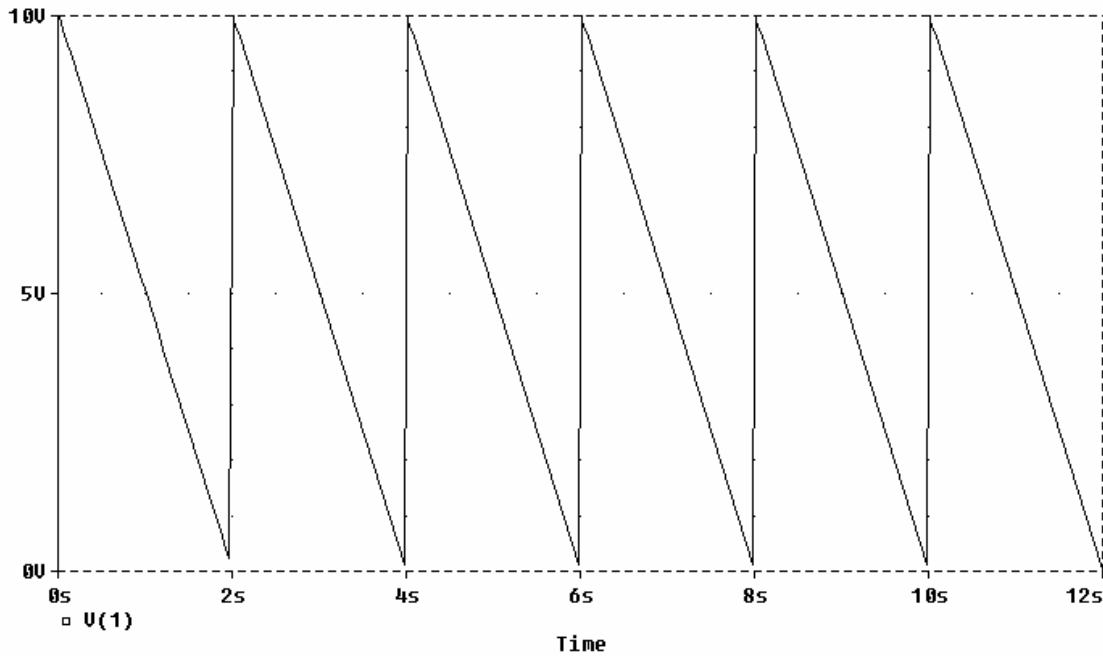
Chapter 17, Solution 67.

The Schematic is shown below. In the Transient dialog box, we type "Print step = 0.01s, Final time = 36s, Center frequency = 0.1667, Output vars = v(1)," and click Enable Fourier. After simulation, the output file includes the following Fourier components,

V1=0

V2=4

TD=1

TR=1

TF=1

PW=2

PER=6

Prob. 16.51.

FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

DC COMPONENT =   2.000396E+00

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	1.667E-01	2.432E+00	1.000E+00	-8.996E+01	0.000E+00
2	3.334E-01	6.576E-04	2.705E-04	-8.932E+01	6.467E-01
3	5.001E-01	5.403E-01	2.222E-01	9.011E+01	1.801E+02
4	6.668E-01	3.343E-04	1.375E-04	9.134E+01	1.813E+02
5	8.335E-01	9.716E-02	3.996E-02	-8.982E+01	1.433E-01
6	1.000E+00	7.481E-06	3.076E-06	-9.000E+01	-3.581E-02
7	1.167E+00	4.968E-02	2.043E-02	-8.975E+01	2.173E-01
8	1.334E+00	1.613E-04	6.634E-05	-8.722E+01	2.748E+00
9	1.500E+00	6.002E-02	2.468E-02	9.032E+01	1.803E+02

TOTAL HARMONIC DISTORTION =   2.280065E+01 PERCENT

Chapter 17, Solution 68.

The schematic is shown below. We set the final time = 6T=12s and the center frequency = 1/T = 0.5. When the schematic is saved and run, we obtain the Fourier series from the output file as shown below.
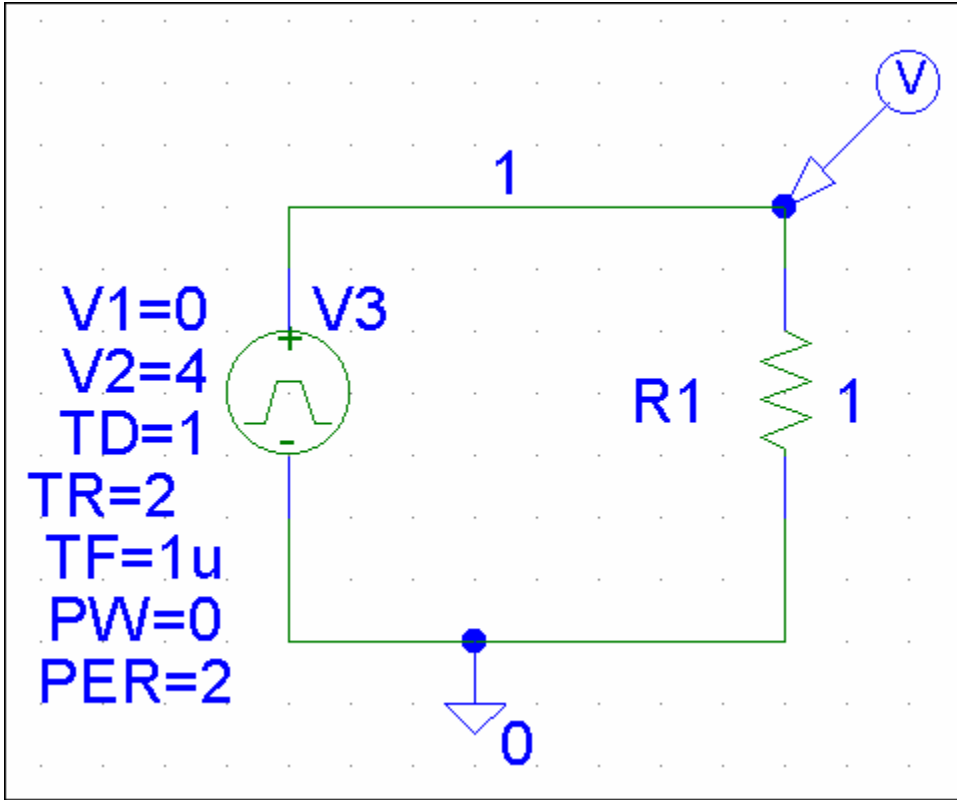

```
FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

 DC COMPONENT =    1.990000E+00
```
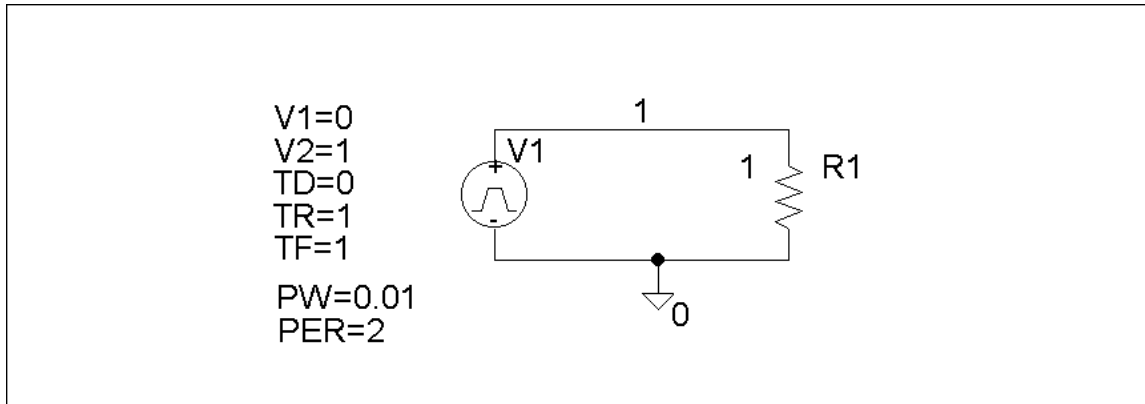
HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	5.000E-01	1.273E+00	1.000E+00	9.000E-01	0.000E+00
2	1.000E+00	6.367E-01	5.001E-01	-1.782E+02	1.791E+02
3	1.500E+00	4.246E-01	3.334E-01	2.700E+00	1.800E+00
4	2.000E+00	3.185E-01	2.502E-01	-1.764E+02	-1.773E+02
5	2.500E+00	2.549E-01	2.002E-01	4.500E+00	3.600E+00
6	3.000E+00	2.125E-01	1.669E-01	-1.746E+0	-1.755E+02
7	3.500E+00	1.823E-01	1.431E-01	6.300E+00	5.400E+00
8	4.000E+00	1.596E-01	1.253E-01	-1.728E+02	-1.737E+02
9	4.500E+00	1.419E-01	1.115E-01	8.100E+00	7.200E+00

Chapter 17, Solution 69.

The schematic is shown below. In the Transient dialog box, set Print Step = 0.05 s, Final Time = 120, Center Frequency = 0.5, Output Vars = V(1) and click enable Fourier. After simulation, we obtain V(1) as shown below. We also obtain an output file which includes the following Fourier components.

FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

DC COMPONENT = 5.048510E-01

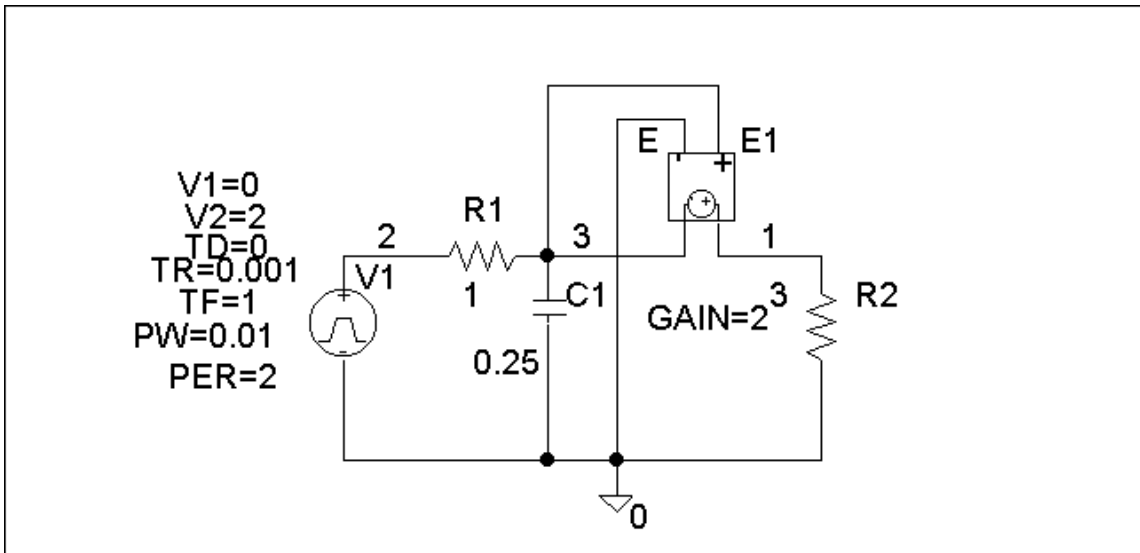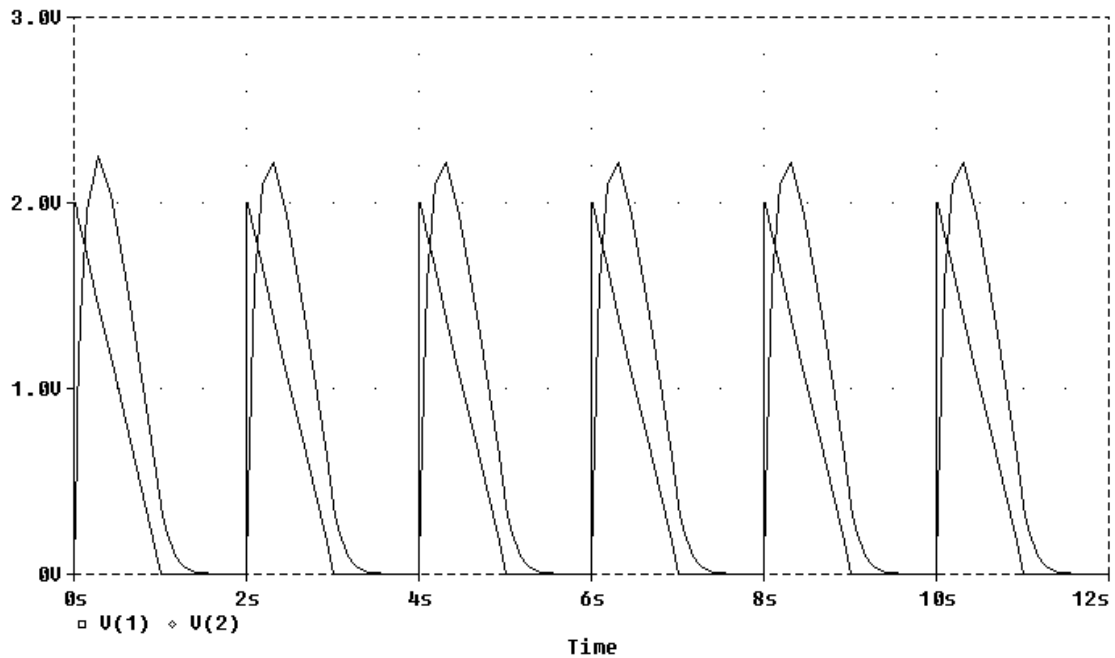| HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	5.000E-01	4.056E-01	1.000E+00	-9.090E+01	0.000E+00
2	1.000E+00	2.977E-04	7.341E-04	-8.707E+01	3.833E+00
3	1.500E+00	4.531E-02	1.117E-01	-9.266E+01	-1.761E+00
4	2.000E+00	2.969E-04	7.320E-04	-8.414E+01	6.757E+00
5	2.500E+00	1.648E-02	4.064E-02	-9.432E+01	-3.417E+00
6	3.000E+00	2.955E-04	7.285E-04	-8.124E+01	9.659E+00
7	3.500E+00	8.535E-03	2.104E-02	-9.581E+01	-4.911E+00
8	4.000E+00	2.935E-04	7.238E-04	-7.836E+01	1.254E+01
9	4.500E+00	5.258E-03	1.296E-02	-9.710E+01	-6.197E+00

TOTAL HARMONIC DISTORTION = 1.214285E+01 PERCENT
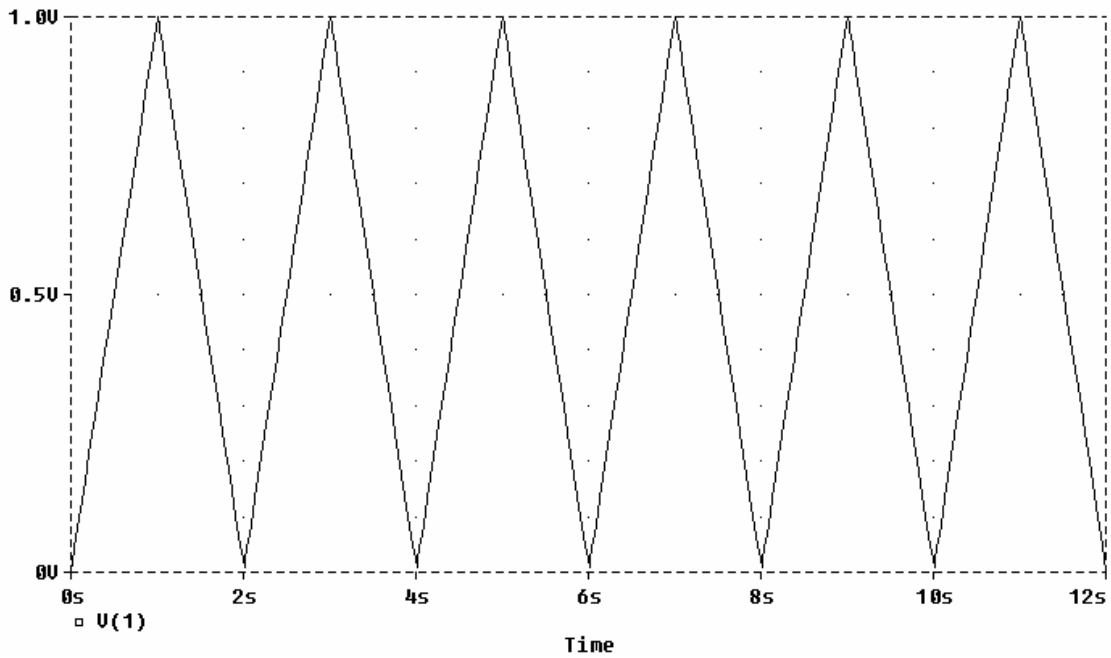
Chapter 17, Solution 70.

The schematic is shown below. In the Transient dialog box, we set Print Step = 0.02 s, Final Step = 12 s, Center Frequency = 0.5, Output Vars = V(1) and V(2), and click enable Fourier. After simulation, we compare the output and output waveforms as shown. The output includes the following Fourier components.

```
FOURIER COMPONENTS OF TRANSIENT RESPONSE V(1)

DC COMPONENT =  7.658051E-01
```
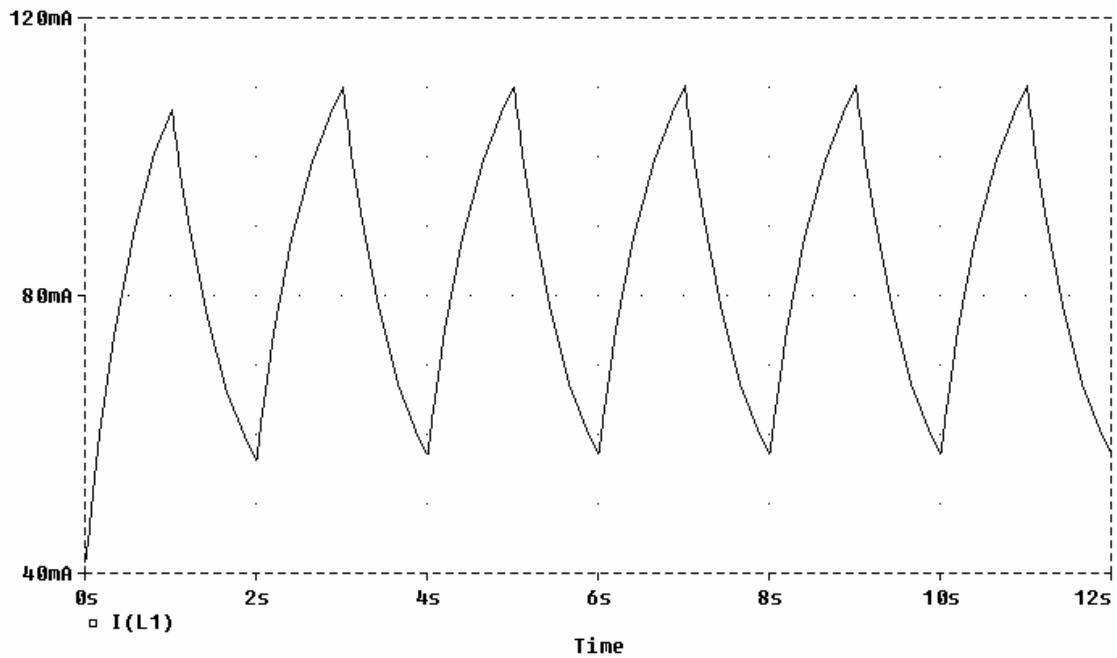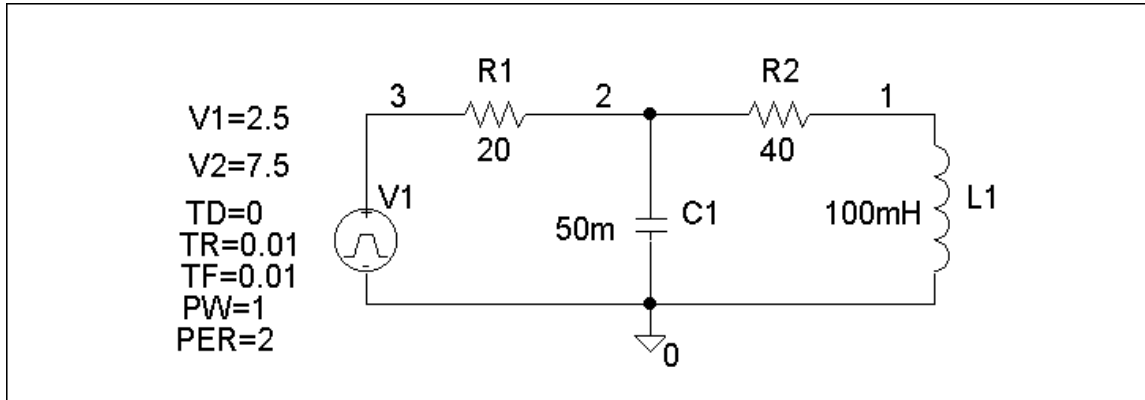
HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	5.000E-01	1.070E+00	1.000E+00	1.004E+01	0.000E+00
2	1.000E+00	3.758E-01	3.512E-01	-3.924E+01	-4.928E+01
3	1.500E+00	2.111E-01	1.973E-01	-3.985E+01	-4.990E+01
4	2.000E+00	1.247E-01	1.166E-01	-5.870E+01	-6.874E+01
5	2.500E+00	8.538E-02	7.980E-02	-5.680E+01	-6.685E+01
6	3.000E+00	6.139E-02	5.738E-02	-6.563E+01	-7.567E+01
7	3.500E+00	4.743E-02	4.433E-02	-6.520E+01	-7.524E+01
8	4.000E+00	3.711E-02	3.469E-02	-7.222E+01	-8.226E+01
9	4.500E+00	2.997E-02	2.802E-02	-7.088E+01	-8.092E+01

```
TOTAL HARMONIC DISTORTION =  4.352895E+01 PERCENT
```

Chapter 17, Solution 71.

The schematic is shown below. We set Print Step = 0.05, Final Time = 12 s, Center Frequency = 0.5, Output Vars = I(1), and click enable Fourier in the Transient dialog box. After simulation, the output waveform is as shown. The output file includes the following Fourier components.

V1=2.5
V2=7.5
TD=0
TR=0.01
TF=0.01
PW=1
PER=2

V1

R1
3   20   2

R2
40   1

50m C1

100mH L1

0

120mA

80mA

40mA

0s   2s   4s   6s   8s   10s   12s
□ I(L1)

Time

FOURIER COMPONENTS OF TRANSIENT RESPONSE I(L_L1)

DC COMPONENT = 8.374999E-02

HARMONIC NO	FREQUENCY (HZ)	FOURIER COMPONENT	NORMALIZED COMPONENT	PHASE (DEG)	NORMALIZED PHASE (DEG)
1	5.000E-01	2.287E-02	1.000E+00	-6.749E+01	0.000E+00
2	1.000E+00	1.891E-04	8.268E-03	8.174E+00	7.566E+01
3	1.500E+00	2.748E-03	1.201E-01	-8.770E+01	-2.021E+01
4	2.000E+00	9.583E-05	4.190E-03	-1.844E+00	6.565E+01
5	2.500E+00	1.017E-03	4.446E-02	-9.455E+01	-2.706E+01
6	3.000E+00	6.366E-05	2.783E-03	-7.308E+00	6.018E+01
7	3.500E+00	5.937E-04	2.596E-02	-9.572E+01	-2.823E+01
8	4.000E+00	6.059E-05	2.649E-03	-2.808E+01	3.941E+01
9	4.500E+00	2.113E-04	9.240E-03	-1.214E+02	-5.387E+01

TOTAL HARMONIC DISTORTION = 1.314238E+01 PERCENT

Chapter 17, Solution 72.

$T = 5, \; \omega_o = 2\pi/T = 2\pi/5$

f(t) is an odd function. $a_o = 0 = a_n$

$$b_n = \frac{4}{T}\int_0^{T/2} f(t)\sin(n\omega_o t)dt = \frac{4}{5}\int_0^{10} 10\sin(0.4n\pi t)dt$$

$$= -\frac{8 \times 5}{2n\pi}\cos(0.4\pi nt)\Big|_0^1 = \frac{20}{n\pi}[1-\cos(0.4n\pi)]$$

$$f(t) = \frac{20}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}[1-\cos(0.4n\pi)]\sin(0.4n\pi t)$$

Chapter 17, Solution 73.

$$p = \frac{V_{DC}^2}{R} + \frac{1}{2}\sum\frac{V_n^2}{R}$$

$$= 0 + 0.5[(2^2 + 1^2 + 1^2)/10] = \underline{\textbf{300 mW}}$$

Chapter 17, Solution 74.

(a) $\quad A_n = \sqrt{a_n^2 + b_n^2}$, $\qquad\qquad \phi = \tan^{-1}(b_n/a_n)$

$\quad A_1 = \sqrt{6^2 + 8^2} = 10$, $\qquad \phi_1 = \tan^{-1}(6/8) = 36.87°$

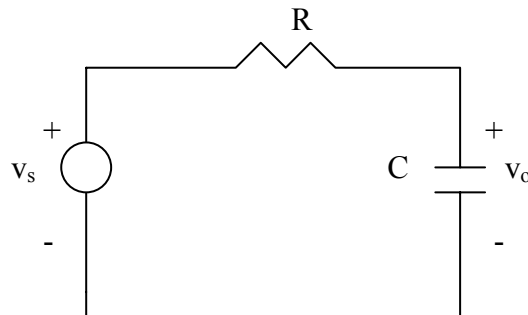$\quad A_2 = \sqrt{3^2 + 4^2} = 5$, $\qquad \phi_2 = \tan^{-1}(3/4) = 36.87°$

$i(t) = \{4 + 10\cos(100\pi t - 36.87°) - 5\cos(200\pi t - 36.87°)\}$ A

(b) $\quad p = I_{DC}^2 R + 0.5\sum I_n^2 R$

$\qquad = 2[4^2 + 0.5(10^2 + 5^2)] = $ **157 W**

Chapter 17, Solution 75.

The lowpass filter is shown below.

$$v_s = \frac{A\tau}{T} + \frac{2A}{T}\sum_{n=1}^{\infty}\frac{1}{n}\sin\frac{n\pi\tau}{T}\cos n\omega_o t$$

$$V_o = \frac{\dfrac{1}{j\omega_n C}}{R + \dfrac{1}{j\omega_n C}}V_s = \frac{1}{1 + j\omega_n RC}V_s, \quad \omega_n = n\omega_o = 2n\pi/T$$

For n=0, (dc component), $V_o = V_s = \dfrac{A\tau}{T}$ $\qquad\qquad\qquad\qquad$ (1)

For the nth harmonic,

$$V_o = \frac{1}{\sqrt{1 + \omega^2_n R^2 C^2} \angle \tan^{-1} \omega_n RC} \cdot \frac{2A}{nT} \sin\frac{n\pi\tau}{T} \angle -90^\circ$$

When n=1, $|V_o| = \dfrac{2A}{T} \sin\dfrac{n\pi\tau}{T} \cdot \dfrac{1}{\sqrt{1 + \dfrac{4\pi^2}{T}R^2C^2}}$ (2)

From (1) and (2),

$$\frac{A\tau}{T} = 50x\frac{2A}{T}\sin\frac{\pi}{10}\frac{1}{\sqrt{1 + \dfrac{4\pi^2}{T}R^2C^2}} \longrightarrow \sqrt{1 + \frac{4\pi^2}{T}R^2C^2} = \frac{30.9}{\tau} = 3.09x10^4$$

$$1 + \frac{4\pi^2}{T}R^2C^2 = 10^{10} \longrightarrow C = \frac{T}{2\pi R}10^5 = \frac{10^{-2}x3.09x10^4}{4\pi x10^3} = \underline{24.59 \text{ mF}}$$

Chapter 17, Solution 76.

$v_s(t)$ is the same as f(t) in Figure 16.1 except that the magnitude is multiplied by 10. Hence

$$v_o(t) = 5 + \frac{20}{\pi}\sum_{k=1}^{\infty}\frac{1}{n}\sin(n\pi t), \quad n = 2k - 1$$

$T = 2, \omega_o = 2\pi/T = 2\pi, \omega_n = n\omega_o = 2n\pi$
$j\omega_n L = j2n\pi; Z = R\|10 = 10R/(10 + R)$
$V_o = ZV_s/(Z + j2n\pi) = [10R/(10R + j2n\pi(10 + R))]V_s$

$$V_o = \frac{10R\angle - \tan^{-1}\{(n\pi/5R)(10 + R)\}}{\sqrt{100R^2 + 4n^2\pi^2(10 + R)^2}}V_s$$

$V_s = [20/(n\pi)]\angle 0^\circ$

The source current I_s is

$$I_s = \frac{V_s}{Z + j2n\pi} = \frac{V_s}{\dfrac{10R}{10 + R} + j2n\pi} = \frac{(10 + R)\dfrac{20}{n\pi}}{10R + j2n\pi(10 + R)}$$

$$= \frac{(10+R)\dfrac{20}{n\pi}\angle - \tan^{-1}\{(n\pi/3)(10+R)\}}{\sqrt{100R^2 + 4n^2\pi^2(10+R)^2}}$$

$$p_s = V_{DC}I_{DC} + \frac{1}{2}\sum V_{sn}I_{sn}\cos(\theta_n - \phi_n)$$

For the DC case, L acts like a short-circuit.

$$I_s = \frac{5}{\dfrac{10R}{10+R}} = \frac{5(10+R)}{10R}, \quad V_s = 5 = V_o$$

$$p_s = \frac{25(10+R)}{10R} + \frac{1}{2}\left[\left(\frac{20}{\pi}\right)^2 \frac{(10+R)\cos\left(\tan^{-1}\left(\frac{\pi}{5}(10+R)\right)\right)}{\sqrt{100R^2 + 4\pi^2(10+R)^2}}\right.$$

$$\left. + \left(\frac{10}{\pi}\right)^2 \frac{(10+R)^2 \cos\left(\tan^{-1}\left(\frac{2\pi}{5}(10+R)\right)\right)}{\sqrt{100R^2 + 16\pi^2(10+R)^2}} + \cdots\right]$$

$$p_s = \frac{V_{DC}}{R} + \frac{1}{2}\sum_{n=1}^{\infty}\frac{V_{on}}{R}$$

$$= \frac{25}{R} + \frac{1}{2}\left[\frac{100R}{100R^2 + 4\pi^2(10+R)^2} + \frac{100R}{100R^2 + 10\pi^2(10+R)^2} + \cdots\right]$$

We want $p_o = (70/100)p_s = 0.7p_s$. Due to the complexity of the terms, we consider only the DC component as an approximation. In fact the DC component has the latgest share of the power for both input and output signals.

$$\frac{25}{R} = \frac{7}{10} \times \frac{25(10+R)}{10R}$$

$$100 = 70 + 7R \text{ which leads to } R = 30/7 = \underline{\textbf{4.286 } \Omega}$$

Chapter 17, Solution 77.

(a) For the first two AC terms, the frequency ratio is $6/4 = 1.5$ so that the highest common factor is 2. Hence $\omega_o = 2$.

$$T = 2\pi/\omega_o = 2\pi/2 = \underline{\pi}$$

(b) The average value is the DC component $= \underline{-2}$

(c) $\qquad V_{rms} = \sqrt{a_o + \dfrac{1}{2}\sum_{n=1}^{\infty}(a_n^2 + b_n^2)}$

$$V_{rms}^2 = (-2)^2 + \frac{1}{2}(10^2 + 8^2 + 6^2 + 3^2 + 1^2) = 121.5$$

$$V_{rms} = \underline{\mathbf{11.02\ V}}$$

Chapter 17, Solution 78.

(a) $\qquad p = \dfrac{V_{DC}^2}{R} + \dfrac{1}{2}\sum\dfrac{V_n^2}{R} = \dfrac{V_{DC}^2}{R} + \sum\dfrac{V_{n,rms}^2}{R}$

$$= 0 + (40^2/5) + (20^2/5) + (10^2/5) = \underline{\mathbf{420\ W}}$$

(b) $\qquad$ 5% increase $= (5/100)420 = 21$

$$p_{DC} = 21\ W = \dfrac{V_{DC}^2}{R} \quad \text{which leads to} \quad V_{DC}^2 = 21R = 105$$

$$V_{DC} = \underline{\mathbf{10.25\ V}}$$

Chapter 17, Solution 79.

From Table 17.3, it is evident that $a_n = 0$,

$$b_n = 4A/[\pi(2n-1)], \quad A = 10.$$

A Fortran program to calculate b_n is shown below. The result is also shown.

```
C       FOR PROBLEM 17.79
            DIMENSION B(20)

            A  =  10
            PIE  =  3.142
            C  =  4.*A/PIE
            DO 10 N = 1, 10
            B(N)  =  C/(2.*FLOAT(N) – 1.)
            PRINT  *, N, B(N)
10          CONTINUE
            STOP
            END
```

n	b_n
1	12.731
2	4.243
3	2.546
4	1.8187
5	1.414
6	1.1573
7	0.9793
8	0.8487
9	0.7498
10	0.67

Chapter 17, Solution 80.

From Problem 17.55,
$$c_n = [1 + e^{-jn\pi}]/[2\pi(1 - n^2)]$$

This is calculated using the Fortran program shown below. The results are also shown.

```
C       FOR PROBLEM 17.80
            COMPLEX X, C(0:20)

            PIE  =  3.1415927
            A  =  2.0*PIE
            DO  10  N = 0, 10
            IF(N.EQ.1) GO TO 10
            X  =  CMPLX(0, PIE*FLOAT(N))
            C(N)  =  (1.0 + CEXP(–X))/(A*(1 – FLOAT(N*N)))
            PRINT *, N, C(N)
10          CONTINUE
            STOP
            END
```
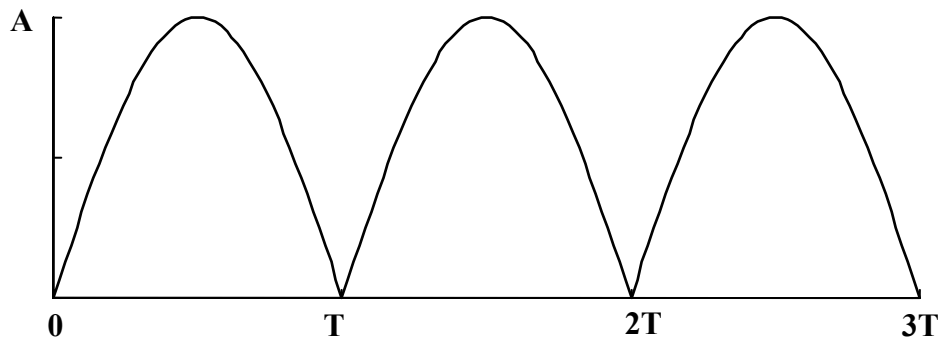
n	c_n
0	$0.3188 + j0$
1	0
2	$-0.1061 + j0$
3	0
4	$-0.2121 \times 10^{-1} + j0$
5	0
6	$-0.9095 \times 10^{-2} + j0$
7	0
8	$-0.5052 \times 10^{-2} + j0$
9	0
10	$-0.3215 \times 10^{-2} + j0$

Chapter 17, Solution 81.

(a)

$$f(t) = \frac{2A}{\pi} - \frac{4A}{\pi} \sum_{n=1}^{\infty} \frac{1}{4n^2 - 1} \cos(n\omega_o t)$$

The total average power is $\quad p_{avg} = F_{rms}{}^2 R = F_{rms}{}^2$ since $R = 1$ ohm.

$$P_{avg} = F_{rms}{}^2 = \frac{1}{T} \int_0^T f^2(t) dt = \underline{\mathbf{0.5A^2}}$$

(b)   From the Fourier series above

$$|c_o| = 2A/2, \quad |c_n| = 4A/[\pi(4n^2 - 1)]$$

n	ω_o	$|c_n|$	$2|c_n|^2$	% power
0	0	$2A/\pi$	$4A^2/(\pi^2)$	81.1%
1	$2\omega_o$	$2A/(3\pi)$	$8A^2/(9\pi^2)$	18.01%
2	$4\omega_o$	$2A/(15\pi)$	$2A^2/(225\pi^2)$	0.72%
3	$6\omega_o$	$2A/(35\pi)$	$8A^2/(1225\pi^2)$	0.13%
4	$8\omega_o$	$2A/(63\pi)$	$8A^2/(3969\pi^2)$	0.04%

(c)   **81.1%**

(d)   **0.72%**

Chapter 17, Solution 82.

$$P = \frac{V_{DC}^2}{R} + \frac{1}{2}\sum_{n=1}^{\infty}\frac{V_n^2}{R}$$

Assuming V is an amplitude-phase form of Fourier series. But

$$|A_n| = 2|C_n|, \quad c_o = a_o$$

$$|A_n|^2 = 4|C_n|^2$$

Hence,

$$P = \frac{c_o^2}{R} + 2\sum_{n=1}^{\infty}\frac{c_n^2}{R}$$

Alternatively,

$$P = \frac{V_{rms}^2}{R}$$

where

$$V_{rms}^2 = a_o^2 + \frac{1}{2}\sum_{n=1}^{\infty}A_n^2 = c_o^2 + 2\sum_{n=1}^{\infty}c_n^2 = \sum_{n=-\infty}^{\infty}c_n^2$$

$$= 10^2 + 2(8.5^2 + 4.2^2 + 2.1^2 + 0.5^2 + 0.2^2)$$

$$= 100 + 2 \times 94.57 = 289.14$$

$$P = 289.14/4 = \underline{\textbf{72.3 W}}$$